MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. XLIX. JANUARY 11, 1889. No. 3

W. H. M. CHRISTIE, M.A., F.R.S., President, in the Chair.

Henry Lord Boulton, Carácas, Venezuela, South America;
Ernest William Brown, B.A., Christ's College, Cambridge, and 16 Milton Terrace, Anlaby Road, Hull;
Samuel Fellows, Tynwald Villas, Lower Villier Street, Wolverhampton;
John James Lewis Goodridge, Bevis Mount, Southampton;
Jesse Scoffin Nimkey, Blanchard House, Whitehall Road, Woodford Wells, Essex;
Frederick William Nash, Holmesdale, Birchfield Road, Birmingham; and
William Schooling, Brechin House, Rivercourt, Hammersmith, W.;

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed as Fellows of the Society, the names of the proposer from personal knowledge being appended:—

The Rev. Charles Douglas Percy Davies, M.A., late Scholar of Pembroke College, Oxford, Ringmer, Lewes (proposed by Professor C. Pritchard); and
Edouard Jules Laumonier, Submarine Telegraph Engineer, Fairfield, Underhill Road, Dulwich (proposed by B. G. Jenkins).

Observations of the Moon, made at the Radcliffe Observatory, Oxford, during the year 1888, *and a Comparison of the Results with the Tabular Places from Hansen's Lunar Tables.* By E. J. Stone, Esq., M.A., F.R.S., Radcliffe Observer.

The present paper contains the right ascensions and north polar distances of the Moon, as deduced from the observations made at the Radcliffe Observatory during the year 1888. These results are here compared with those deduced from Hansen's Lunar Tables on two suppositions :—

(1) That the mean times, found in the usual way from the sidereal times at mean noon, given in the *Nautical Almanac*, were *not* changed in 1864 by the adoption of a different unit of time.

(2) That these mean times *were* changed by the adoption of a different measure of time in 1864 to fix the positions of the clock stars relatively to the Sun, in accordance with the views which I have explained in papers communicated to the Society.

It will be seen that the mean annual error of Hansen's Tables from 1847 to 1863 is $-1''\cdot30$, and that no law of regular increase is apparent ; but that, with the argument in the Tables taken out in the usual way, the mean annual error has since increased at an average rate of $0''\cdot73$ per annum, the error now amounting to about $+17''$. The mean annual error of Hansen's Tables from 1864 to 1888 taken out with the *corrected* argument is $-1''\cdot28$, which is almost identically the same as the mean error between 1847 and 1863.

For facilities for an accurate comparison between Hansen's Lunar Tables and observations we are again indebted to the places published in the *Connaissance des Temps*.

TABLE I.

Radcliffe Observations of the Moon, 1888.

Day, 1888.	Correction to be subtracted from M.T. for change of Sidereal to Time at Mean Noon since 1864.	Observer.	Observed R.A.	Seconds of Hansen's R.A.	Hansen *minus* Observed. **Uncorrected** for Error in Time.	Correction due to Error in Time.	Hansen *minus* Observed. **Corrected** for Error in Time.	Observed N.P.D.	Seconds of Hansen's N.P.D.	Hansen *minus* Observed. **Uncorrected** for Error in Time.	Correction due to Error in Time.	Hansen *minus* Observed. **Corrected** for Error in Time.
			h m s	s	s	s	s	° ′ ″	″	″	″	″
Jan. 23	35·73	W.	4 4 24·84	25·71	+0·87	−1·22	−0·35	74 10 47·65	43·82	−3·83	+4·11	+0·28
26	35·74	W.	6 48 16·55	17·53	+0·98	−1·39	−0·41	69 32 20·94	19·90	−1·04	−0·04	−1·08
27	35·75	R.	7 46 56·48	57·56	+1·08	−1·41	−0·33	70 11 1·40	3·02	+1·62	−1·82	−0·20
Feb. 1	35·77	F. B.	12 35 18·51	19·73	+1·22	−1·33	−0·11	88 55 49·42	51·87	+2·45	−7·44	−4·99
25	35·86	F. B.	9 16 22·84	24·13	+1·29	−1·42	−0·13	73 29 3·61	7·46	+3·85	−4·47	−0·62
Mar. 21	35·96	F. B.	6 52 24·05	24·98	+0·93	−1·35	−0·42	69 20 57·12	56·55	−0·57	−0·19	−0·76
April 2	36·01	R.	18 44 41·76	43·27	+1·51	−1·45	+0·06	110 45 14·50	14·28	−0·22	−0·04	−0·26
26	36·11	W.	15 8 42·41	43·93	+1·52	−1·50	+0·02	102 35 43·30	48·49	+5·19	−6·68	−1·49
30	36·12	R.	19 22 9·40	10·98	+1·58	−1·47	+0·11	110 46 27·72	26·63	−1·09	+1·14	+0·05
May 18	36·19	R.	9 54 29·76	30·34	+0·58	−1·33	−0·75	75 9 13·40	15·29	+1·89	−5·25	−3·36
21	36·21	F. B.	12 40 2·47	3·40	+0·93	−1·36	−0·43	89 15 19·02	23·48	+4·46	−7·91	−3·45
23	36·21	F. B.	14 36 24·76	25·94	+1·18	−1·47	−0·29	100 0 58·25	64·43	+6·18	−7·34	−1·16
25	36·22	R.	16 42 21·76	23·00	+1·24	−1·57	−0·33	108 17 46·27	49·71	+3·44	−4·29	−0·85
June 18	36·32	W.	13 13 59·98	60·80	+0·82	−1·35	−0·53	92 21 41·73	45·68	+3·95	−7·78	−3·83
19	36·32	R.	14 10 14·58	15·27	+0·69	−1·39	−0·70	97 37 23·86	28·23	+4·37	−7·49	−3·12

Day, 1888.	Correction to be subtracted from M.T. for change of Sidereal Time at Mean Noon since 1864.	Observer.	Observed R.A.	Seconds of Hansen's R.A.	Hansen *minus* Observed. **Uncorrected** for Error in Time.	Correction due to Error in Time.	Hansen *minus* Observed. **Corrected** for Error in Time.	Observed N.P.D.	Seconds of Hansen's N.P.D.	Hansen *minus* Observed. **Uncorrected** for Error in Time.	Correction due to Error in Time.	Hansen *minus* Observed. **Corrected** for Error in Time.
	s		h m s	s	s	s	s	° ′ ″	″	″	″	″
June 25	36·35	W.	20 27 4·56	6·20	+1·64	−1·48	+0·16	109 31 4·40	2·25	−2·15	+3·14	+0·99
July 20	36·45	R.	17 50 16·57	17·65	+1·08	−1·55	−0·47	110 34 15·22	16·22	+1·00	−2·06	−1·06
23	36·46	W.	20 58 8·47	10·09	+1·62	−1·44	+0·18	108 15 52·84	49·30	−3·54	+4·00	+0·46
26	36·47	R.	23 39 44·55	45·72	+1·17	−1·21	−0·04	96 44 42·23	35·31	−6·92	+6·64	−0·28
Aug. 18	36·56	F. B.	19 32 12·19	13·54	+1·35	−1·50	−0·15	110 56 17·92	15·30	−2·62	+1·40	−1·22
22	36·58	W.	23 17 35·47	36·80	+1·33	−1·25	+0·08	98 40 38·67	32·64	−6·03	+6·54	+0·51
23	36·58	F. B.	0 7 6·04	7·47	+1·43	−1·20	+0·23	94 7 48·19	42·59	−5·60	+6·84	+1·24
Sept. 12	36·66	W.	17 9 15·87	17·28	+1·41	−1·50	−0·09	109 32 49·47	52·99	+3·52	−3·32	+0·20
13	36·67	R.	18 10 59·82	61·03	+1·21	−1·51	−0·30	111 7 28·98	31·07	+2·09	−1·28	+0·81
17	36·68	W.	22 5 23·75	25·01	+1·26	−1·32	−0·06	104 24 9·69	6·15	−3·54	+5·49	+1·95
19	36·69	W.	23 47 30·73	31·97	+1·24	−1·21	+0·03	95 55 47·98	41·58	−6·40	+6·79	+0·39
20	36·69	R.	0 35 40·96	42·03	+1·07	−1·18	−0·11	91 17 25·74	19·02	−6·72	+6·92	+0·20
Oct. 11	36·78	R.	18 53 52·68	54·03	+1·35	−1·53	−0·18	111 37 59·57	58·13	−1·44	+0·15	−1·29
15	36·79	F.	22 41 10·66	11·91	+1·25	−1·26	−0·01	101 51 49·32	43·71	−5·61	+6·11	+0·50
17	36·80	R.	0 19 3·11	4·21	+1·10	−1·17	−0·07	92 56 59·91	53·89	−6·02	+6·93	+0·91
20	36·81	F. B.	2 39 27·46	28·65	+1·19	−1·17	+0·02	79 31 13·66	10·15	−3·51	+6·01	+2·50

Day, 1888.	Correction to be subtracted from M.T. for change of Sidereal Time at Mean Noon since 1864.	Observer.	Observed R.A.	Seconds of Hansen's R.A.	Hansen *minus* Observed. **Uncorrected** for Error in Time.	Correction due to Error in Time.	Hansen *minus* Observed. **Corrected** for Error in Time.	Observed N.P.D.	Seconds of Hansen's N.P.D.	Hansen *minus* Observed. **Uncorrected** for Error in Time.	Correction due to Error in Time.	Hansen *minus* Observed. **Corrected** for Error in Time.
	s		h m s	s	s	s	s	° ′ ″	″	″	″	″
Oct. 22	36·82	R.	4 16 12·52	13·62	+1·10	−1·23	−0·13	72 34 58·28	55·64	−2·64	+4·16	+1·52
23	36·83	F. B.	5 6 41·72	42·92	+1·20	−1·27	−0·07	70 11 21·64	21·15	−0·49	+2·92	+2·43
Nov. 13	36·91	F. B.	0 4 30·61	31·85	+1·24	−1·18	+0·06	94 32 61·26	55·75	−5·51	+6·92	+1·41
16	36·92	F. B.	2 23 49·37	50·51	+1·14	−1·16	−0·02	80 54 38·65	33·76	−4·89	+6·29	+1·40
17	36·93	R.	3 11 0·90	1·87	+0·97	−1·19	−0·22	76 55 30·80	25·21	−5·59	+5·57	−0·02
20	36·94	F. B.	5 41 11·13	12·37	+1·24	−1·30	−0·06	68 55 28·07	24·66	−3·41	+2·05	−1·36
25	36·96	R.	10 7 55·77	56·87	+1·10	−1·31	−0·21	75 12 20·37	23·11	+2·74	−5·61	−2·87
27	36·97	R.	11 53 32·93	33·98	+1·05	−1·32	−0·27	84 14 52·81	59·08	+6·27	−7·67	−1·40
Dec. 7	37·01	R.	21 8 54·40	55·66	+1·26	−1·47	−0·21	103 29 54·49	50·07	−4·42	+4·51	+0·09
11	37·02	F. B.	0 36 21·22	22·43	+1·21	−1·17	+0·04	91 34 52·32	47·89	−4·43	+7·07	+2·64
12	37·03	R.	1 22 42·37	43·21	+0·84	−1·15	−0·31	86 54 23·45	18·24	−5·21	+6·90	+1·69
13	37·03	F. B.	2 8 46·53	47·89	+1·36	−1·15	+0·21	82 25 12·29	7·93	−4·36	+6·50	+2·14
26	37·08	W.	13 19 22·37	23·70	+1·33	−1·33	0·00	92 41 23·28	30·97	+7·69	−8·03	−0·34
Mean of Errors, without regard to sign			...	...	s 1·185	...	s 0·204	...	...	″ 3·830	...	″ 1·348
Mean Errors for Year ..			...	...	+1·185	...	−0·149					

Observers: W., Mr. W. Wickham; R., Mr. W. H. Robinson; F. B., Mr. F. A. Bellamy.

TABLE II.

Radcliffe Observations of the Moon, 1888.

Errors of the Moon's Tabular Place in Longitude and Ecliptic Polar Distance. **Uncorrected** *and* **Corrected** *for the change in Tabular Mean Time introduced in the year* 1864.

Day, 1888.		Errors of Longitude (Hansen *minus* Observed).		Errors of E.N.P.D. (Hansen *minus* Observed	
		Uncorrected.	**Corrected.**	**Uncorrected.**	**Corrected.**
		″	″	″	″
Jan.	23	+13·11	− 5·03	−1·33	−0·70
	26	+13·64	− 5·83	−2·17	−0·60
	27	+15·32	− 4·63	−1·12	+0·63
Feb.	1	+17·89	− 3·50	−4·95	−3·95
Feb.	25	+18·86	− 1·97	−1·90	−0·03
March	21	+12·98	− 5·95	−1·74	−0·23
April	2	+21·14	+ 0·86	+1·40	−0·19
April	26	+22·93	− 0·12	−1·08	−1·52
	30	+22·15	+ 1·52	+2·03	+0·27
May	18	+ 8·56	−11·38	−1·08	+0·54
	21	+14·67	− 7·33	−1·37	−0·64
	23	+18·57	− 4·45	+0·46	+0·23
	25	+17·99	− 4·78	+1·06	−0·22
June	18	+12·92	− 8·84	−1·00	−0·53
	19	+11·19	−10·91	+0·65	+0·57
	25	+23·03	+ 1·96	+3·47	+1·50
July	20	+15·20	− 6·63	+0·73	−0·94
	23	+23·14	+ 2·33	+3·03	+1·16
	26	+18·80	− 0·44	+0·57	−0·49
Aug.	18	+19·11	− 1·88	+0·35	−1·53
	22	+20·54	+ 0·89	+2·19	+0·93
	23	+21·98	+ 2·68	+3·40	+2·52

Day, 1888.		Errors of Longitude (Hansen *minus* Observed).		Errors of E.N.P.D. (Hansen *minus* Observed).	
		Uncorrected.	**Corrected.**	**Uncorrected**	**Corrected.**
Sept.	12	+20″·22	− 1″·25	+1″·80	+0″·31
	13	+16·90	− 4·21	+2·39	+0·73
	17	+18·41	− 1·50	+3·10	+1·52
	19	+19 60	+ 0·25	+1·50	+0·54
	20	+17·50	− 1·60	+0·13	−0·47
Oct.	11	+18·90	− 2·38	+0·32	−1·52
	15	+19·14	− 0 32	+1·69	+0·41
	17	+17·64	− 1·33	+1·01	+0·42
	20	+17·85	− 0·49	+2·03	+2·47
	22	+15·99	− 2·10	+0·15	+1·17
	23	+16·93	− 1·22	+1·07	+2·33
Nov.	13	+19·29	+ 0·26	+2·34	+1·65
	16	+17·64	− 0·74	+0 83	+0·23
	17	+15·21	− 3·10	−1·58	−0·88
	20	+17·48	− 0·80	−2·86	−1·39
	25	+15·92	− 3·87	−3·04	−1·61
	27	+16·94	− 4·27	−0·50	+0·32
Dec.	7	+18·44	− 2·88	+1·00	−0·79
	11	+18·55	− 0·49	+3·07	+2 67
	12	+13·67	− 4·96	−0·11	−0·17
	13	+20·59	+ 2·22	+2·75	+3·07
	26	+21·46	− 0·13	−0 35	−0·31
Mean of Errors, without regard to sign		**...17″·682**	**3″·052**	**1″ 607**	**1″·020**
Mean Errors for Year		**+17″·682**	**−2″·462**		

TABLE III.

Mean Excess over Observation of the Moon's Tabular Place in Longitude for the years 1847 *to* 1888, *as computed from Hansen's Tables.*

Uncorrected *and* **Corrected** *on and after* 1864 *for the change in Tabular Mean Time introduced in the year* 1864.

Year.	Errors of Longitude (Hansen *minus* Observed). **Uncorrected.**	**Corrected.**
	″	″
1847	+ 1·07	+ 1 07
1848	+ 0·20	+ 0·20
1849	− 0·47	− 0·47
1850	− 0·28	− 0·28
1851	− 1·29	− 1·29
1852	− 0·92	− 0 92
1853	− 1·63	− 1·63
1854	− 1·68	− 1·68
1855	− 0·87	− 0 87
1856	− 0·96	− 0·96
1857	− 1·86	− 1·86
1858	− 1·98	− 1·98
1859	− 1·80	− 1 80
1860	− 2·90	− 2·90
1861	− 2·19	− 2·19
1862	− 2·83	− 2·83
1863	− 1·61	− 1·61
1864	+ 0·12	− 0·81
1865	+ 1·27	− 0·22
1866	+ 2·14	− 0·22
1867	+ 3·48	+ 0·36
1868	+ 4·12	+ 0·28
1869	+ 4·28	− 0·35
1870	+ 4·83	− 0 66
1871	+ 6 96	+ 0·44
1872	+ 7·31	+ 0·10
1873	+ 8·24	+ 0·20
1874	+ 9·29	+ 0 56
1875	+ 9·87	+ 0·36
1876	+ 9 80	− 0·50
1877	+ 9·23	− 1 90
1878	+ 8·22	− 3·60
1879*	+ 9·63	− 3·12
1880	+ 10·89	− 2·77
1881	+ 10·51	− 4·06
1882†	+ 12·68	− 2·51
1883	+ 14·71	− 1·50
1884	+ 14·65	− 1·91
1885	+ 15·20	− 1·82
1886	+ 15·34	− 2·53
1887	+ 15·70	− 3·25
1888‡	+ 17·68	− 2 46

* All to 1879, Greenwich observations.
† 1880 to 1882, Mean of Greenwich and Radcliffe.
‡ 1883 and since, Radcliffe only.

Radcliffe Observatory, Oxford:
1889, *January* 9.

On the Determination of Errors of Graduation without Cumulative Error, and the Application of the Method to the Scales of the Cape Heliometer. By David Gill, LL.D., F.R.S., Her Majesty's Astronomer at the Cape of Good Hope.

The methods available for determining the accidental errors of graduation of a scale or circle may be divided into two classes—

1. The methods in which the error of successive subdivision is cumulative.

2. Those methods in which this error is non-cumulative.

The first of these, which at best is weak and unsatisfactory, has hitherto been almost exclusively employed; whilst the second method has been rarely, and then only partially, used.

The only instance which occurs to me of the employment of the second method in ordinary astronomical practice is in the first stage of the investigation of the division errors of a Transit Circle. The operation may serve for illustration. Let us suppose that, as in the Greenwich and Cape instruments, the graduated circle is read by six microscopes, which we shall call A, B, C, D, E, F, and that the true angular distances between the zeros of these microscopes are respectively $60° + d_b$, $60° + d_c$, $60° + d_d$, $60° + d_e$, $60° + d_f$, $60° + d_a$ (when $60° + d_b$ represents the angle between microscopes A and B, and so on), and that the true lengths of the six fundamental arcs of the circle are $60° + e_{60}$, $60° + e_{120}$, $60° + e_{180}$, $60° + e_{240}$, $60° + e_{300}$, $60° + e_{360}$, then the errors E of divisions will be—

$$\left.\begin{aligned} E_{60} &= e_{60} \\ E_{120} &= e_{60} + e_{120} \\ E_{180} &= e_{60} + e_{120} + e_{180} \\ E_{240} &= e_{60} + e_{120} + e_{180} + e_{240} \\ E_{300} &= e_{60} + e_{120} + e_{180} + e_{240} + e_{300} \\ E_{360} &= e_{60} + e_{120} + e_{180} + e_{240} + e_{300} + e_{360} \end{aligned}\right\} \quad \ldots\ldots (1)$$

It is evident also that we have

$$d_a + d_b + d_c + d_d + d_e + d_f = 0$$

and

$$e_{60} + e_{120} + e_{180} + e_{240} + e_{300} + e_{360} = 0 \quad \ldots\ldots (2)$$

Therefore if we compare the arc $(60° + e_{60})$ successively with each of the six microscope intervals, we obtain

e_{60} = mean excess of this arc over mean of micrometer intervals,

with the probable error

$$\pm \frac{\epsilon}{\sqrt{6}}$$

where ϵ is the probable error of one comparison.

Similarly, we may determine each of the quantities e_{120}, e_{180}, . . . e_{360}, with similar probable errors.

But when we come to the computation of the division errors E_{60}, E_{120}, &c., by our formula (1) we shall probably find that our equation (2) is not satisfied; indeed the probability is that the sum of the corrections will differ from 0 by $\pm\epsilon$.

This, therefore, represents the method in which the process of subdivision creates cumulative error.

But if instead of observing only the space 0° to 60°, we read all the microscopes whilst division 0 is brought successively under all the microscopes, we automatically satisfy our equation (2) as well as our equation (1), and the process is not affected by cumulative error (since any error of reading which would increase the apparent length of one space would diminish the apparent length of the adjoining space), so long as it can be assumed that the intervals between the microscopes remain unchanged. This condition can be practically realised by repeating the order of observation in the opposite sense.

The general condition which alone secures freedom from cumulative error is

For a circle.—*There shall be as many microscopes (or the equivalent for as many microscopes) as the number of spaces into which the circle is to be subdivided.*

If it is required to subdivide a straight line or the portion of a circle into n spaces (i.e. when the beginning and ending of scale are not identical), the number of the microscopes (or their equivalent) must be $n+1$.

Thus, if it were required to determine the division errors of a circle to each single degree without cumulative error, it would obviously be impossible to provide 360 microscopes for the purpose, but it is comparatively easy to provide an equivalent arrangement. In the Transit Circle made by T. Cooke & Sons, of York, for Mr. Newall under the superintendence and direction of Mr. Marth, two concentric circles are provided, one of which is firmly bolted to the axis, whilst the second circle may be rotated with respect to the first. The graduated surfaces of both circles are in the same plane, and nearly contiguous, so that the graduations of both circles may be read off by the same microscope. The operation of determining the division error of each degree of both circles may then be carried out without cumulative error, as follows.

The movable circle is turned relative to the fixed circle till the divisions marked 0° on both circles are coincident; the movable circle is then clamped. Each degree of the circle is then brought successively under the microscope, and the divisions of both circles are read. When this has been done the movable circle is unclamped and turned till its graduation marked 0 coincides with the division marked 1° on the fixed circle,

and all the corresponding divisions on both circles are again read. The same operation is then repeated after shifting the movable circle so that its division marked 0 coincides with that marked 2° on the fixed circle, and so on until each degree-space of the fixed circle has been compared with each degree-space of the movable circle. The mean excess of any degree-space of the fixed circle over the mean of the 360 degree-spaces of the movable circle is then, obviously, the excess of the length of that nominal degree over a true degree.

In this way the true length of each nominal degree-space of the fixed circle is directly obtained, with a mean error

$$= \pm \frac{\epsilon}{\sqrt{360}},$$

where E is the mean error of a single comparison of a degree-space on the two circles. Further, the nature of the process automatically fulfils the condition that the sum of the degree-spaces so obtained is = 0, and that there is no cumulative error in the formation of the division errors by successive summations of the errors of the single degree-spaces, so that all the division errors are determined with the same accuracy—viz. with a mean error

$$= \pm \frac{\epsilon}{\sqrt{360}}.$$

The method is perfected if instead of one microscope two microscopes 180° apart are employed, so that the errors of *diameters* are determined with complete elimination of possible changes of eccentricity of the movable circle. The method may be obviously extended to the determination of each single subdivision of the degree-spaces. So far as I am aware, however, the instrument designed by Mr. Marth, although it was made, has never been used for astronomical purposes, and we have no records of the results of the investigation of division errors by the method in question.

But I was so much impressed by the strength of Mr. Marth's proposed method and by the inefficiency of previously existing methods for investigating the errors of graduation of heliometer scales, that I determined, if possible, in the new Cape heliometer to provide means for determining the division errors of the scales by some method which would permit the absolute error of each division to be known with the utmost accuracy with which the scales can be read.

In vol. ii. of the Dun Echt publications, p. 48, it is shown that although the principal divisions of the Dun Echt heliometer could be determined with a probable error less than $\pm 0''{\cdot}02$, the probable error of the determination of the errors of graduation amounted in some parts of the scale to $\pm 0''{\cdot}1$ (a quantity greater than the probable error of a single complete observation), and

which no amount of observation would sensibly diminish, because at p. 50 of the same work it is further shown:

1. There is a limited accuracy in the determination of the error of each division, beyond which no amount of observation will strengthen the result.
2. This limit of accuracy is different for different divisions, and seems to depend on the perfection of the surface and the cleanness of the cut.
3. For any division the limit of accuracy is from one-third to one-fourth of the probable error of a single careful bisection.

It is therefore useless to trust to the mean of a very large number of observations to reduce the accumulation of errors which must arise in the process of successive subdivision, and it is of course hopeless to attempt to reach results of the highest accuracy by any process except one in which the errors are non-cumulative.

In order to secure divisions with very clean-cut edges, I proposed to Messrs. Repsold that they should, if possible, avoid the usual process of "charcoaling" the scales after division. This process consists in rubbing the divisions with a stick of fine charcoal to remove the "burr" at the edge of the divisions; but it does so at the expense of widening the cut of the division, and destroys the absolute cleanness of the narrow, deep furrow which is cut out by a well-selected diamond, the cutting front and edge of which have been produced by cleavage. At first I thought that it might be better to leave the "burr" rather than destroy the perfection of the division by the charcoaling process, but Messrs. Repsold pointed out that if the "burr" were not removed it would not be possible to clean the scales by the usual plan of wiping with silk or soft linen, and the "burrs" would catch in the texture of the cleaning-material, and possibly be liable when so rubbed to change the appearance of the division under the microscope. I then suggested the possibility of polishing down the entire surface of the scale with a small flat metal polisher charged with some grinding- or polishing-material. If the divisions are cut sufficiently deep, this plan would enable the artist gradually to remove not only the entire surface affected by the "burr," but to regulate the equality of the width of the divisions along the entire scale. Messrs. Repsold undertook the construction of the scales on this plan, and one member of the firm devoted three months to experimental work and to the completion of the exquisite scales which are fitted to the Cape heliometer. These scales are made of an alloy of iridium and platinum, with polished surfaces, and the divisions are so fine as to be almost invisible to the naked eye. Each scale is one decimetre in length, divided into 200 parts of 0·5 millimetre each. Four revolutions of the micrometer correspond with one division,

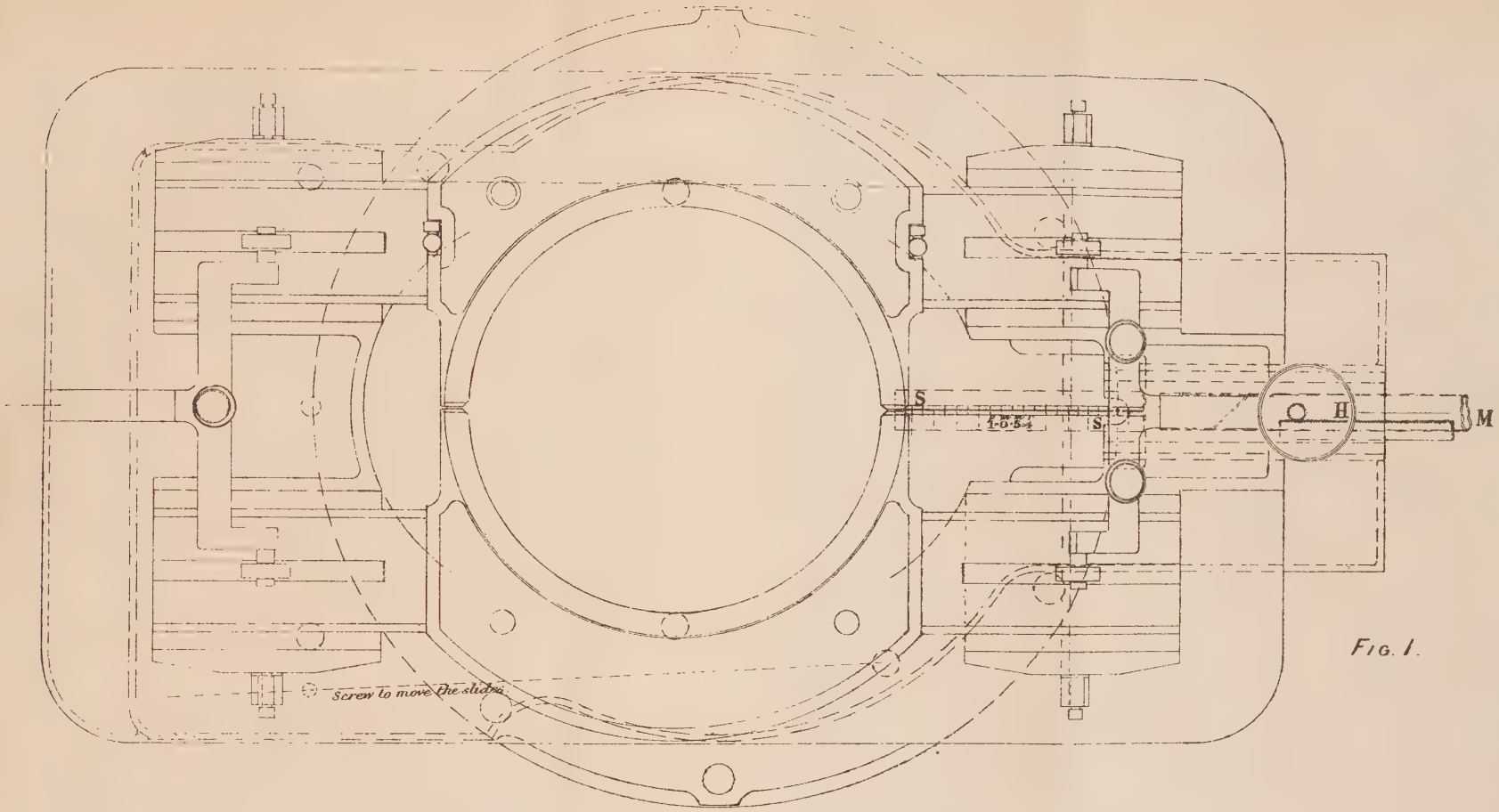

S
1·0·5·4
S
H
M
Screw to move the slides

FIG. 1.

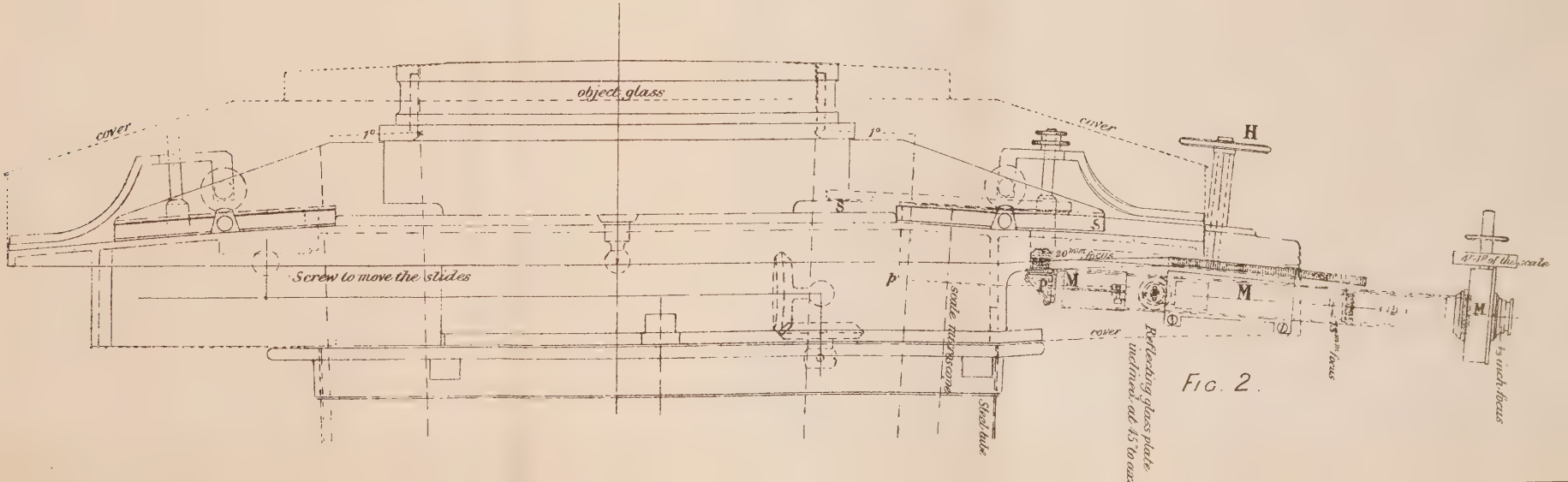

object glass
cover
1°
1°
cover
H
S
S
Screw to move the slides
P
20mm focus
P
M
M
cover
Reflecting glass plate inclined at 45 to axis
Steel tube
4th of the scale
M
to each focus

FIG. 2.

and one revolution is equivalent to 10″ of arc.* The microscope magnifies 100 diameters, and yet, under this unusual power, the divisions, when illuminated by light reflected from a dead white surface, present sharp clean edges, such as I have never before seen in like perfection on any other scales or divided circles which I have had the opportunity of using.

When the image of a division is placed symmetrically between the parallel spider-webs of the micrometer a clean sharp white line appears between the inner edge of either web and the division, and the eye can judge so sharply of the equality of the thickness of these white lines that it is seldom that two consecutive pointings on the division differ by more than $\frac{1}{10}$th part of a division of the micrometer head, i.e. by 0″·01, which is entirely satisfactory.

In order to permit the comparison of any part of one scale with any part of the other, I suggested to Messrs. Repsold that the reading microscope should be so arranged as to be capable of being directed to any part of the scales. It soon became evident on discussion that, to fulfil this condition, it would be necessary to increase very materially and inconveniently the diameter of the central cylinder which rotates in the cradle, and of the tube generally, and consequently to diminish the ease of rotation. After some discussion Messrs. Repsold proposed the plan which entirely avoids this, and which was ultimately approved and carried out as represented in the accompanying sketches.

Fig. 1 represents the "head" of the heliometer removed from the tube and viewed as if from the eye-end.

Fig. 2 represents a section of the head through the axis in a plane passing through the line of section of the object glass. S S, figs. 1 and 2, represent the scales.

From the eye-end of the telescope these scales are read by a long fixed microscope, of which the axis is indicated by the dotted line in fig. 2, which is marked "scale microscope."

For determining the errors of division of the scale the auxiliary microscope M M M is employed. This microscope slides in cylindrical bearings, of which the axis passes through the focal point of the object glass at right angles to the plane represented in fig. 2. In other words, the cylindrical bearings in which the division-error micrometer slides are planed and ground simultaneously with, and from the same centre as the bearings in which the mountings of the two segments of the telescopic object glass slide. The microscope is moved in the direction of the length of the scale by means of a rack and pinion motion, which is actuated by the milled head H. The scale is in the principal focus of an achromatic object glass of 20 mm. focal length, which renders parallel rays passing from

* More rigorously, one reckoning = 10″·024. The round number 10″ is retained throughout this paper as sufficiently exact for our purposes.

the scale through the prism of total reflection P; these rays are received by an achromatic objective of 73 mm. focus, which forms an enlarged image of the scale in the plane of the wires of the micrometer. Illumination of the scale is effected by reflection of the light of a small electric glow-lamp (passed through ground glass) from the surface of an elliptical reflector of parallel transparent glass which is placed between the prism and the objective of 73 mm. focus.

By turning the milled head H the observer can view a range of the scale $=\frac{2}{3}$ of its total length—that is, from the position of the prism P as represented in fig. 2 (in the position where the effective graduation begins) to the position of the same prism indicated by the dotted line *p*. This range permits the comparison of any part of one half of each scale with any part of either half of the other, and this, as will be afterwards shown, permits the complete determination of the errors of division of both scales.

The further details of the figures do not relate to the special problem in question, and are, besides, probably sufficiently obvious without further description.

In order to illustrate the actual *modus operandi*, let us suppose for simplicity that it is required to find the division errors of two similar scales, each divided into three parts. Let these divisions be numbered 0, 1, 2, 3 on one scale, and 10, 11, 12, 13 on the other.

Fig. *a*. Fig. *b*. Fig. *c*. Fig. *d*. Fig. *e*.

The operation then divides itself into five series of observations.

For the first series the scales are moved from the position of fig. *c* (i.e. from the position of coincidence of the optical centres of the object glass) to the position shown in fig. *a*. Then, by means of the milled head H, the centre of the micrometer objective is brought over the divisions 0 and 12, and the readings of the micrometer, when division 0 and 12 are successively bisected, are taken; then, *without moving the relative position of the segments*, the micrometer objective is brought over the divisions 1 and 13 and similar readings are made.

Let m_{a0}, m_{a12}, m_{a1}, m_{a13} represent the micrometer readings on the divisions 0, 12, 1, and 13 respectively in series *a*, and let us suppose that the readings of the micrometer head increase as the webs move in the direction of increasing readings of the scale. Then we have

Excess of the length of the space (13−12) over the length of the space (1−0) ... ... $= (m_{a13} - m_{a12}) - (m_{a1} - m_{a0}) = n_{a1}$.

Next, the scales are set as in fig. *b*, and by quite a similar method of proceeding we get

$$\left.\begin{array}{l}\text{Excess of the length of the space (12−11) over the length of the space (1−0)} \\ \text{... ... ...}\end{array}\right\} = (m_{b12} - m_{b11}) - (m_{b1} - m_{b0}) = n_{b1}$$

$$\left.\begin{array}{l}\text{Excess of the length of the space (13−12) over the length of the space (2−1)} \\ \text{... ... ...}\end{array}\right\} = (m_{b13} - m_{b12}) - (m_{b2} - m_{b1}) = n_{b2}$$

and so on similarly through the series *c*, *d*, and *e*, until each scale interval from 0 to 3 has been compared with each scale interval from 10 to 13.

The results may then be conveniently arranged as follows:—

$$\left.\begin{array}{lccc} & (13-12) & (12-11) & (11-10) \\ (1-0) & n_{a1} & n_{b1} & n_{c1} \\ (2-1) & n_{b2} & n_{c2} & n_{d1} \\ (3-2) & n_{c3} & n_{d2} & n_{e1} \end{array}\right\} \quad \ldots\ldots \quad (3)$$

If we denote the excess of the length of the scale (3–0) over the mean length of the two scales by $\Delta\alpha$, and the excess of (13–10) over the same mean by $\Delta\beta$ we obtain the length of each space as follows:—

$$\Delta\beta - \Delta\alpha = n_{a1} + n_{b2} + n_{c3} + n_{b1} + n_{c2} + n_{d2} + n_{c1} + n_{d1} + n_{e1} \quad \ldots \quad (4)$$
$$\Delta\beta + \Delta\alpha = 0$$

$$\left.\begin{array}{ll} \Delta(11-10) = \frac{n_{c1} + n_{d1} + n_{e1}}{3} + \frac{\Delta\alpha}{3} & \Delta(1-0) = -\frac{n_{a1} + n_{b1} + n_{c1}}{3} + \frac{\Delta\beta}{3} \\ \Delta(12-11) = \frac{n_{b1} + n_{c2} + n_{d2}}{3} + \frac{\Delta\alpha}{3} & \Delta(2-1) = -\frac{n_{b2} + n_{c2} + n_{d1}}{3} + \frac{\Delta\beta}{3} \\ \Delta(13-12) = \frac{n_{a1} + n_{b2} + n_{c3}}{3} + \frac{\Delta\alpha}{3} & \Delta(3-2) = -\frac{n_{c3} + n_{d2} + n_{e1}}{3} + \frac{\Delta\beta}{3} \end{array}\right\} \quad \ldots \quad (5)$$

of which the sums give our equation (4).

The corrections for division error are then

$$\left.\begin{array}{l} \text{For division—} \\ 11 = \Delta(11-10) \quad \ldots \\ 12 = \Delta(11-10) + \Delta(12-11) \quad \ldots \\ 13 = \Delta(11-10) + \Delta(12-11) + \Delta(13-12) \\ \text{For division—} \\ 1 = \Delta(1-0) \quad \ldots \\ 2 = \Delta(2-1) + \Delta(1-0) \quad \ldots \\ 3 = \Delta(3-2) + \Delta(2-1) + \Delta(1-0) \quad \ldots \end{array}\right\} \quad \ldots\ldots \quad (6)$$

and these corrections are determined without cumulative error, because all the side equations are rapidly satisfied by the conditions of measurement.

The effective range of scale A of the Cape heliometer is graduated from 10^d to 190^d, and that of scale B from 210^d to 390^d, so that there is never any ambiguity as to the relative positions of the segments, or possibility of mistake as to which scale was read.

As the scale-division microscope has (from structural necessity) a range only equal to two-thirds of the length of the scale, it is only possible to compare the lower half of either scale with both halves of the other scale; and the scales were for purposes of examination considered to consist of four sections.

$$\text{Scale A} \begin{cases} \alpha = 10 \text{ to } 100 \\ \beta = 100 \text{ to } 190 \end{cases}$$

$$\text{Scale B} \begin{cases} \gamma = 210 \text{ to } 300 \\ \delta = 300 \text{ to } 390 \end{cases}$$

We can then compare any part of α with any part of γ and δ, and any part of γ with any part of α and β, but we cannot compare β and δ directly.

A complete solution is afforded by comparing

All subdivisions of α with all subdivisions of δ

" " γ " " β

The total length of α with the total length of γ

and this method gives all the division errors with equal weight with the minimum of labour.*

But it would try the observer's eyesight too severely to compare all the ninety subdivisions of α with the ninety subdivisions of γ at a single sitting (corresponding to series c in the previous example), nor could one rely on the constancy of the relative positions of the two slides between two or more sittings. I therefore determined, in the first place, to investigate only the errors of each 10th division, leaving the subdivisions of these intervals for investigation by subsequent series of measures.

The comparison of all subdivisions of α with those of δ, and of γ with those of β, were each made by four independent and complete series of observations, and the total length of α with that of γ was also determined by four sets of observations.

The reductions were carried out on the plan described above, with the mean results which are given in the following table. In order to enable the reader to estimate the accuracy of these mean results, the comparison of the mean result with each of the four independent determinations is given for each division.

* If we compare also all parts of α with all parts of γ we obtain the errors of the subdivisions of α and γ with double weight, and we cannot compare β and δ directly in order similarly to increase the weight of the determination of the subdivisions of these sections.

TABLE A.

	Division	Division Error from Mean of 4 Series.	Comparison of the Result of each Independent Series with the Mean.			
		″	″	″	″	″
	10	0·000	...	...	...	...
	20	+ ·067	− ·028	+ ·012	+ ·003	+ ·013
	30	+ ·083	− ·018	± ·000	− ·006	+ ·024
	40	+ ·238	− ·009	+ ·009	− ·002	+ ·002
α	50	+ ·146	− ·005	+ ·005	− ·002	+ ·002
	60	+ ·048	− ·002	− ·002	± ·000	+ ·004
	70	+ ·076	− ·018	− ·004	+ ·005	+ ·017
	80	+ ·203	− ·001	− ·006	+ ·008	− ·001
	90	+ ·057	− ·017	+ ·007	+ ·004	+ ·006
	100	+ ·043	...	...	...	...
	110	+ ·036	− ·028	− ·005	+ ·019	+ ·014
	120	+ ·036	− ·016	+ ·004	± ·000	+ ·012
	130	− ·026	− ·018	+ ·009	+ ·009	± ·000
	140	− ·025	− ·027	+ ·003	+ ·008	+ ·016
β	150	+ ·093	− ·019	− ·008	− ·001	+ ·028
	160	+ ·078	− ·008	− ·007	− ·003	+ ·018
	170	+ ·054	− ·013	+ ·020	− ·007	+ ·001
	180	+ ·057	− ·015	− ·002	− ·003	+ ·014
	190	+ ·105	...	...	...	...
	210	0·000	...	...	...	...
	220	+ ·021	− ·009	− ·005	+ ·011	+ ·003
	230	+ ·051	+ ·012	+ ·001	− ·019	+ ·006
	240	+ ·131	+ ·009	+ ·007	− ·011	− ·005
	250	+ ·016	− ·011	+ ·001	+ ·021	− ·011
γ	260	− ·133	− ·014	− ·005	+ ·023	− ·004
	270	− ·060	− ·014	+ ·005	+ ·011	− ·002
	280	+ ·021	+ ·008	− ·005	+ ·010	− ·013
	290	− ·105	+ ·012	+ ·003	± ·000	− ·015
	300	− ·134	...	...	...	...
	310	− ·142	+ ·016	− ·001	− ·007	− ·008
	320	− ·125	+ ·026	− .009	− ·019	+ ·002
	330	− ·156	+ ·009	± ·000	− ·010	± ·000
	340	− ·168	+ ·022	+ ·010	− ·024	− ·008
δ	350	− ·073	− ·006	+ ·016	− ·015	+ ·005
	360	− ·098	+ ·015	+ ·013	− ·014	− ·014
	370	− ·177	+ ·023	+ ·010	− ·016	− ·016
	380	− ·157	+ ·009	+ ·027	− ·019	− ·019
	390	− ·105	...	...	...	...

With these definitive results the original observations are represented as follows :—

TABLE B.

Comparison of α and δ. (*Excess of δ.*)

(O − C)

	390–380	380–370	370–360	360–350	350–340	340–330	330–320	320–310	310–300
	"	"	"	"	"	"	"	"	"
20–10	+0·003	−0·017	+0·013	+0·041	+0·002	−0·018	−0·014	−0·009	−0·003
30–20	− ·017	− ·009	− ·021	− ·017	+ ·007	+ ·017	+ ·018	− ·001	+ ·008
40–30	+ ·024	− ·027	− ·012	− ·039	− ·004	− ·009	+ ·017	− ·031	− ·010
50–40	− ·006	+ ·041	+ ·015	− ·017	− ·021	− ·018	+ ·029	+ ·007	− ·029
60–50	− ·025	+ ·017	− ·002	+ ·018	+ ·011	− ·021	− ·021	+ ·032	− ·007
70–60	− ·011	− ·010	− ·015	+ ·007	+ ·006	+ ·004	− ·002	+ ·018	+ ·007
80–70	+ ·040	+ ·013	+ ·004	+ ·018	+ ·002	− ·032	− ·041	− ·002	− ·006
90–80	− ·017	− ·020	− ·005	− ·005	+ ·022	+ ·043	− ·007	− ·020	+ ·017
100–90	− ·002	+ ·003	+ ·016	− ·018	− ·031	+ ·024	+ ·006	− ·001	+ ·007

Comparison of β and γ. (*Excess of β.*)

(O − C)

	190–180	180–170	170–160	160–150	150–140	140–130	130–120	120–110	110–100
	"	"	"	"	"	"	"	"	"
220–210	−0·009	−0·026	−0·008	+0·029	−0·010	+0·026	+0·014	±0·000	−0·019
230–220	+ ·009	+ ·022	− ·022	+ ·020	− ·004	+ ·037	− ·036	− ·005	− ·011
240–230	− ·001	+ ·014	− ·020	+ ·006	− ·042	+ ·020	+ ·011	+ ·009	± ·000
250–240	± ·000	− ·007	− ·017	− ·006	+ ·007	± ·000	+ ·010	− ·003	+ ·021
260–250	+ ·015	+ ·014	+ ·014	− ·002	− ·009	− ·030	+ ·013	− ·020	+ ·005
270–260	− ·023	− ·012	+ ·010	− ·008	+ ·007	+ ·006	+ ·035	+ ·006	− ·002
280–270	+ ·014	− ·025	+ ·009	+ ·011	+ ·006	− ·045	− ·001	+ ·024	+ ·006
290–280	− ·013	+ ·014	+ ·004	− ·024	+ ·050	− ·014	− ·023	+ ·009	− ·001
300–290	+ ·016	+ ·010	+ ·026	− ·019	− ·007	− ·003	− ·029	− ·019	+ ·022

The probable error of the definitive results may be deduced either from the residuals of Table A or of Table B.

In the first case, the square of the *mean error* of the determination of a division error by a single series comes out

0·000182;

whence the *probable error* of the determination of each division from the mean of the four series is

± 0″·0044.

Secondly, take the mean of the squares of the residuals in Table B, and divide this by nine, and we have the *mean error* of the determination of the length of each interval, and therefore,

if the errors are non-cumulative, this corresponds also with the *mean error* of division error, and accordingly we find the corresponding probable error to be also

$$\pm 0''{\cdot}0044.$$

This affords a very crucial practical proof of the non-cumulative character of the errors of the process.

A less crucial but more evident proof is afforded by a glance at Table A, which shows that the four independent determinations of the middle divisions of the sections (50 or 60; 140 or 150; 250 or 260; 340 or 350) agree as well together as do the four independent determinations of division error near the beginning and end of α, β, γ, δ.

The linear value corresponding to the angular probable error is

$$0{\cdot}000055 \text{ millimetre} = \frac{1}{500{,}000} \text{ inch (in round numbers).}$$

Such accuracy is the best proof of the exceeding sharpness of the divisions and the high perfection of the micrometric apparatus generally.

In the investigation of the errors of the single divisions now in progress, I content myself with two series of observations for the subdivision of each 10-division space. This will afford a probable error of

$$\pm 0''{\cdot}0062$$

for the subdivision, or of

$$\sqrt{0{\cdot}0044^2 + 0{\cdot}0062^2} = \pm 0''{\cdot}0076$$

for the probable error of the determination of the absolute division error of each single division.*

It may be contended that such accuracy is in excess of practical requirements; this question can only be considered by discussing the results of actual observations.

I have selected for the purpose of this discussion my measures of the distance of two stars from β *Centauri*, because they are all made within a comparatively short period of time and under a greater variety of hour-angle than those of any of the other stars which I am observing for parallax, except such as have been observed at periods six months apart; and as none have yet been observed at three such periods,† I have no means, therefore, of eliminating from the observations the effects of parallax and proper motion.

* The work, continued for an hour each day (it is undesirable, on account of the strain on the eyes, to work longer at a stretch, on division errors), requires about nine months for its completion, but it is a labour from which I do not shrink, in view of the importance of the determination in connection with the series of determinations in which the instrument is employed.

† The definitive researches on parallax were only begun on February 25, when the electric light arrangements were finally fitted.

The comparison stars are —

	Mag.	1875. R.A. h m s	Decl. ° ′
γ = Gould's Zones XIII. 2817	8	13 46 0	−60 43
δ = „ „ XIV. 258	8	14 3 56	−58 48

The stars are situated very symmetrically with respect to β *Centauri* both in distance and position-angle, and are therefore very suitable for discussing the probable error of observation as distinguished from that error affected by changes in the scale value. The following are the results of all the measures in question, converted into arc, from the scale and screw readings, with an assumed (approximately correct) constant screw value, and corrected only for refraction and aberration :—

1888.	Hour Angle. h m	Temperature.	γ	δ	$p-q$
		°	″	″	°
June 7	+4 40	58 F.	5045·27	5380·05	137
8	−2 52	57	45·05	80·14	296
12	−2 37	56	45·02	79·95	292
17	−2 5	55	45·10	80·01	281
17	+4 2	50	45·15	80·13	145
18	−2 24	55	44·84	79·71	287
18	+3 48	50	44·88	79·68	148

The column marked $p-q$ gives the angle which the line joining the stars makes with the vertical at the mean epoch of observation, where p is the position-angle of the star γ with respect to β *Centauri*, and q the parallactic angle.

The sums and differences of the measured distances are

	$\delta+\gamma$	v	$\delta-\gamma$	v
	″	″	″	″
June 7	10425·32	+0·32	334·78	−0·13
8	5·19	+ ·19	5·09	+ ·18
12	4·97	− ·03	4·93	+ ·02
17	5·11	+ ·11	4·91	± ·00
17	5·28	+ ·28	4·98	+ ·07
18	4·55	− ·45	4·87	− ·04
18	4·56	− ·44	4·80	− ·11
Mean	10425·00		334·91	

and

$$\frac{vv}{n-1} = 0\cdot1043 \text{ for } \delta+\gamma\text{; and } 0\cdot0114 \text{ for } \delta-\gamma\text{;}$$

hence the probable error of one observation is

For $\delta+\gamma$ ± 0″·218

For $\delta-\gamma$ ± 0″·072.

Of course in all refined observations it is only the *differences* that are employed, or else some other equivalent method is used to eliminate small inevitable changes in the scale value, due partly, perhaps, to inequalities of the temperature of different parts of the instrument, partly to physiological changes in the normal adaptation of the focus of the observer's eye.

Now a probable error of $\pm 0''\cdot072$ in the measurement of the difference of the distances means a very high accuracy indeed in the determination of parallaxes, for the *difference* of the distances is affected by twice the parallax.

Suppose that a pair of comparison stars are found nearly in the major axis of the parallactic ellipse, and that four observations only are made, arranged at the times most suitable for parallax determination. These observations afford the equations

$$\begin{aligned} a - 0\cdot5\Delta\mu - 2\pi &= n_1 \\ a \qquad\quad + 2\pi &= n_2 \\ a \qquad\quad + 2\pi &= n_3 \\ a + 0\cdot5\Delta\mu - 2\pi &= n_4. \end{aligned}$$

Where a is a constant, $\Delta\mu$ the error of the assumed proper motion, and π the stars' parallax.

Their solution by least squares gives π with a weight$=16$, and therefore with a probable error

$$= \frac{\pm 0''\cdot072}{\sqrt{16}} = \pm 0''\cdot018$$

from *only four* observations.

If the observations are reduced in the same manner as is adopted by Professor Pritchard at Oxford, viz. to apply corrections proportional to the observed distances, such that the sum of the distances observed on the same day is a constant, then the following results are obtained:—

	γ	v	δ	v
	"	"	"	"
June 7	5045·11	+0·06	5379·89	−0·06
8	4·96	− ·09	80·04	+ ·09
12	5·03	− ·02	79·97	+ ·02
17	5·05	·00	79·95	·00
17	5·01	− ·04	79·99	+ ·04
18	5 06	+ ·01	79·94	− ·01
18	5·10	+ ·05	79·90	− ·05
Mean	5045·05		5379·95	

Reduced in this method (which is the method adopted by Professor Pritchard when he compares the Oxford photographic measures with heliometer observations) the *probable error of a single heliometer measure* is

$$\pm 0''\cdot035,$$

or about one-third of the probable error of the best Oxford photographic results.

That such accuracy can be attained in the single measurement is sufficient argument for insisting on accuracy to $\pm 0''.01$, in the determination of the division errors, where the final result may rest not only upon a single observation but on many repeated observations in which the same divisions are employed.

The whole of the observations quoted in the preceding paper have been made by myself, but some independent measures of the same errors have been made by Mr. Finlay, which show no sensible difference in his results from mine. Indeed I am now convinced that the personality frequently found in the readings of scales and circles is due to the ragged character of the edges of the divisions; and, if in future the original graduations are made with a *cutting* diamond, and the "burrs" are removed by the process above described (instead of by the method of charcoaling), and if like linear perfection of the edges of the divisions is attained, personality in reading will disappear. I do not mean that personality will disappear in the *absolute* readings (for an observer may have a tendency to place the division too much to the right or left of the middle point between the parallel wires), but that it will disappear from the *difference* of the readings on two divisions.

It may be noted, finally, that the observations of β *Centauri* above quoted, as well as other researches which will afterwards be published, afford proof of the complete elimination of systematic error depending on the direction of measurement with respect to the vertical, which is secured by the use of the reversing prism.

Royal Observatory, Cape of Good Hope:
1888, *October* 15.

On Methods of printing Stellar Charts from Photographic Negatives. By Isaac Roberts.

Since the introduction of the "Stellar Pantograver" at the meeting of this Society in November last I have taken my negative of the 1,270 polar stars, which formed one of the illustrations to the descriptive matter concerning the instrument to Col. Sir C. W. Wilson, the director of the Ordnance Survey at Southampton, and explained to him the difficulties attendant upon the transference of the images of very faint stars from negatives on to paper so as to form reliable charts; and he readily and cordially undertook to try if the methods in use for printing the maps of the Government Survey would give satisfactory results with stellar photographs. The two sheets of illustrations which I present to the Society, and which are now before us, show the fine work that Sir C. W. Wilson has been able to execute from the negative by

his printing processes, and the results can now be compared with those obtained by aid of the pantograver.

There are eleven illustrations on each sheet, which may be described thus:—Five silver bromide prints, which were exposed to gaslight for the following intervals respectively: 20, 25, 30, 35, and 60 seconds each. Five platinotype prints, which were exposed to diffused sunlight for 3, 4, 5, 20, and 30 minutes respectively; and one photozincograph.

On the other sheet are five silver bromide prints reversed, the stars being black on a white ground. The duration of the exposures to gaslight were 1, 2, 3, 4, and 5 minutes respectively. Five platinotype prints, reversed like the bromide prints, and exposed to diffused sunlight for 25, 30, 35, 40, and 45 minutes respectively. One photozincograph, also reversed.

All the prints are remarkably sharp and clear, and obviously represent a state of perfection in instrumental, chemical, and manipulative skill that experience and ample resources can alone supply, and on the lines of the methods here illustrated it is not probable that more can be done than is here shown. Let us then compare the results with those obtained by the aid of the pantograver, and this can best be done by counting all the stars that are with certainty recognisable as such, without a magnifier, on each illustration; but, to shorten the process, I have counted the stars on three of the plates which show the largest number, with the following results: 452, 379, and 436 stars. I have also attached to the sheets two copies of the stars which were engraved with the pantograver, so that eye comparisons can readily be made. It will be evident on inspection, without the trouble of counting, that very large numbers of stars are missing from each print, and many others are reduced so that they cannot with certainty be identified as stars, and it would be impossible to measure with accuracy the stellar magnitudes on any of the prints. If any speck on the film of a negative should resemble a star it would of course print so that it could not be distinguished from a star; and on most negatives there are such specks more or less numerous.

If now we compare the foregoing results with those of the method of engraving we find the following advantages in favour of the latter:—

1. In the method of engraving, a magnifying power of about twenty-four diameters is used, and with it specks on the film are readily and with certainty distinguished from stars. The prints will therefore be free from these sources of error.

2. The engraving-instrument is made to proportionally *increase* the star diameter so that the faintest image on the film can be made visible to sight on the copper plate and on the print from it without altering the position of the centre. Accurate measurements of both positions and magnitudes of the stars can therefore be made from the engravings.

3. No star, however faint its image may be on the film, is lost on the engraving, except through the carelessness of the operator. In the comparison we are now making with the photographic prints, which have been made, as already stated, with the highest practical skill on zinc plates, nearly two-thirds of the faint stars are lost on a single plate which covers a sky space of two degrees in diameter, and many others are so small that we are in some doubt if they really are stars, and it is hopeless to try to determine their magnitudes with accuracy.

The conclusion, then, to which we are led at present is, that any known system of charting the stars directly from the original negatives by the aid of photographic prints fails to meet the essential conditions in number of the stars charted—in the mensurability of their magnitudes, and in freedom from liability of printing specks as stars; whereas the proposed system of engraving meets fully each condition and leaves the original negative uninjured for future reference.

Photographs of the Nebulæ in the Pleiades and in Andromeda.
By Isaac Roberts.

The accompanying photograph* of the nebulæ in the *Pleiades* is an enlargement to four times the negative, taken on December 8, 1888, with exposure of the plate during four hours, and it brings to view all the nebulosity shown on the photographs which I have presented to the Society, and referred to at the meeting in January 1887 in a manner more clearly defined, though not exceeding the limits shown on those photographs. The nebulosity involves the following stars: *Asterope*, *Taygeta*, *Maia*, *Celæno*, *Electra*, *Alcyone*, stars 24,† 12, and far towards the *sp* of *Merope*. *Alcyone* is seen in the midst of a dense spiral nebula which branches in *np* direction towards and up to stars 9 and 8; and towards the north there are two faint circles of nebulous matter, one with its centre *np* star 15 and the other with its centre about midway between *Alcyone* and star 24. It is cut by a long nebulous streamer which extends both *p* and *f* star 24 and forms the chord of an arc. *Merope* is involved in dense streaky nebulosity, which extends far away in *sp* direction, where it fades gradually into the darkness of space. There is a crooked nebulous bridge connecting the *Merope* and *Alcyone* nebulæ, which passes over star 13. A nebulous straight line commences at star 7 and extends through star 1 in *p* direction till past the centre of *Electra*, where it ends. *Electra* has a horn-like projection from it, pointed towards *Alcyone*. *Maia* is involved in dense streaky nebulosity which extends to *Taygeta* and *Asterope* with a detached branch towards

* The photographs are placed in the Library. † Bessel's numbers.

and involving star 12, and also a horn-like projection in the direction of star *l*. There are also streaky masses of nebulosity trending north and south, filling the space between *Maia*, *Electra*, and *Merope*. There is a remarkable absence of symmetry between these nebulæ, that leads us to infer that we are looking at a number of separate nebulæ one behind the other in the line of sight, and two of them, which the nebulous straight lines represent, are seen edgewise. Many stars are visible through even the densest nebulosities.

We shall henceforward be able to study these objects by aid of reliable data when verbal or written descriptions will be better understood in giving the results of generalisations.

Nebula in Andromeda.

The accompanying photograph was taken on December 29, 1888, and is enlarged to three times the negative. The exposure was during four hours, and it confirms the various details shown upon the first photograph, which was presented to the Society at the last meeting, besides bringing to sight more clearly, by reason of the longer exposure, details that were either faint or absent upon the first. I am engaged in measuring the distances and angles between certain stars and the nuclei of the three nebulæ, together with the positions of some of the well-defined bright parts of the rings of the great nebula, so that in future any movement, orbital or otherwise, in their relative positions may be detected. When the measurements are completed they shall be presented to the Society.

The Surface of the Sun in 1888. By the Rev. S. J. Perry.

A few words will suffice to tell the history of solar changes during the past twelve months. More free from spots than in any previous year of the present decade, the Sun has still shown some signs of vigour and activity. On 241 days of the year observations were secured at Stonyhurst, and out of these the surface was found to be entirely free from spots on 102 days, and even faculæ were totally absent on 6 days. Throughout the year the faculæ were generally scarce and faint, but, when the definition was fairly good, it seldom happened that a few scattered bright markings could not be detected, although it might sometimes require a careful search to find them. Occasionally, however, a few bright scattered faculæ near the poles were the most striking feature of the Sun's surface. Comparing this year with those immediately preceding it, we find the percentage of spotless days to be greatly on the increase, having been 9·42 in 1886, 29·73 in 1887, and 42·32 in 1888. The greatest number of consecutive days during this minimum period, on which the spot-area never exceeded the one hundred thousandth part of the

visible hemisphere, was from May 24 to August 27, 1888, and the next longest succession of almost spotless days occurred during the autumn of 1886. In the month of January the solar disturbances consisted of small round spots of the normal type, but for the rest of the year small groups were more common than isolated spots, the principal member of the group being almost invariably followed by a number of lesser companions. In general, the spots did not last long, being for the most part very shallow disturbances.

Watching the general surface on days when the definition is unusually perfect, there does not seem to be much change from year to year, nor is there even a very marked difference between the appearance at a maximum and at a minimum period. The veiled spots continue to show themselves with great persistency in every portion of the surface, and the character and rapidity of their changes appear to be unaffected by the causes that produce the variations in the solar cycle. The sub-permanent veiled-spots, seen only within the spot-zones, have not been recorded more than twice or thrice during the last twelve months; but the slight penumbral markings, which start as ill-defined dots, and, quickly spreading out, congregate in considerable numbers so as to form large blurred patches, have often been seen most distinctly. The darkness of the shade in these markings is sometimes much more intense than usual, and not unfrequently these darker-veiled spots are observed in large numbers in the neighbourhood of scattered faculæ.

An observation of some importance was made on the day preceding the closing day of the year, and which I may perhaps be allowed here to recall, as it seems to indicate the commencement of a fresh solar cycle. The observation was the record of a small group of spots in the high latitude of 36° S, whereas spots have of late been confined within a rather narrow equatorial zone. If this group proves to be the forerunner of other spots similarly situated, we have fair grounds for concluding that a continual decrease in the mean spot-area, until the end of the present year, will be followed in 1890 by a rapid increase in the number and extent of solar disturbances.

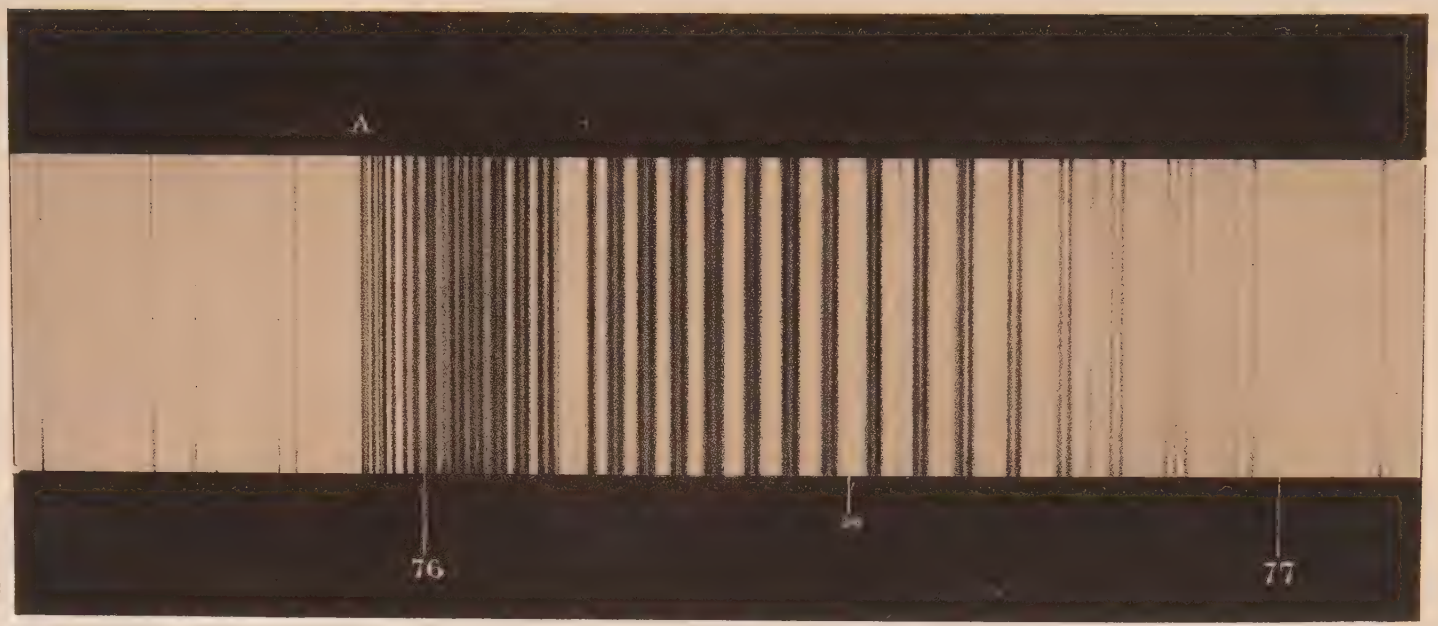
Stonyhurst Observatory:
1889, *January* 9.

Photographs of the Red End of the Solar Spectrum from the Line (D) to the Line (A) in Seven Sections. Taken by F. McClean, M.A., F.R.A.S.

The accompanying photographs represent the red end of the solar spectrum from Fraunhofer line (D) to Fraunhofer line (A). They comprise just one half of the visible spectrum from the line (H) to the line (A).

Dr. Rowland's published photographs of the solar spectrum

THE A GROUP of the SOLAR SPECTRUM



Photographed by J. McClean.

W.H. Wesley lith. West, Newman & Co imp.

(excluding the ultra-violet spectrum beyond the line H) comprise the portion of the spectrum between wave-lengths 3,900 and 5,800 tenth metres (10^{-10} metres). The present photographs comprise from wave-length 5,800 (above D) to wave-length 7,700 (below A), or just as much again.

The sections, numbered from VII. to XIII., into which the photographs are divided, correspond to the sections of Ångström's normal solar spectrum—counting from the section containing (H) towards the red.

The scale of Section VII. (containing the line D) is the same as the same section of Ångström's Chart, but from this onwards the scale increases slightly along with the corresponding increase of the dispersion of the spectroscope employed. The spectroscope employed consists of a Rutherford Grating, ruled 17,296 lines to the inch, and about $1\frac{3}{4}$ inch square. The grating rotates, and the telescopes are fixed. The dispersion, therefore, increases as the secant of the angle of displacement of the grating from its central position. The increase in the scale of the photographs is about one-tenth from (D) to (A).

The red spectrum photographed is that of the second order. But besides the principal photographs of the sections of this spectrum, subsidiary photographs are given, showing, in the same sections, both the red spectrum of the second order as before, and also the overlapping, green to violet, spectrum of the third order. These double spectra were taken successively through the upper and lower portions of the same slit, and they serve as tests of the freedom of the red spectrum from the overlapping third order spectrum.

The positions of the leading divisions of the scale of wave-lengths are taken from Ångström's Chart. From (D) to (C) the scale is placed directly on the second order spectrum, while from (C) to (A) the scale is first applied to the subsidiary third order spectrum, and that of the second order spectrum obtained from it. Since the wave-lengths, at common points of the overlapping spectra, are strictly in the ratio of 3 to 2, these double spectra thus furnish an accurate scale to the obscurer parts of the red spectrum.

The present photographs are enlarged about $8\frac{1}{2}$ times from the original negatives. It must be remembered that the photographs of the green to violet portion of the third order spectrum, included in the subsidiary photographs, are not given as finished photographs of that portion of the spectrum. They are only given for the special purposes explained above. Their definition has, so far, suffered in the endeavour to obtain the two overlapping spectra on the same photograph. Also, the object-glasses at present employed in the spectroscope fail in bringing to a definite focus the image of the spectrum situated above the line G.

The principal photographs also need improvement in many respects, and they are only put forward in their present state

because they form, substantially, complete photographs of the red end of the solar spectrum such as have not hitherto been produced.

This paper is illustrated by a very faithful drawing by Mr. Wesley of the A group of lines, taken from the photographs. The photographs themselves have been placed in the Library.

1888, *December* 13.

Note on Observations of Nebulæ Spectra at Hurstside Observatory.

By Albert Taylor.

The observations of nebulæ spectra recorded in this paper have been made at Sir Henry Thompson's Observatory (recently erected at Hurstside, West Molesey, Surrey) with a 12-inch refractor by Cooke, and a new star spectroscope by Hilger, specially adapted for observations of spectra of faint stars and nebulæ. The dispersive apparatus consists of a train of one 60° prism and two half-prisms, the half-prisms being attached to the lenses of the collimator and observing-telescope of the spectroscope. The micrometer is constructed so that one turn of the screw moves the cross-wires $\frac{1}{100}$th of an inch, and the drum being divided into 100 parts, readings can be made with accuracy to $\frac{1}{10000}$th of an inch, and by estimation to a smaller fraction than that.

The cross-wires of the micrometer can be illuminated at will by means of a small electric lamp. The eyepiece usually used for measurements has a power of 15, but a power of 10 was used for the nebula in *Lyra*.

The nebulæ examined were the Great Nebulæ in *Orion* and *Andromeda* and the Ring Nebula in *Lyra*.

The Great Nebula in Orion.

The spectrum of this nebula has been observed on every possible occasion during the last four months of 1888, and the results, while fully confirming Dr. Copeland's 'Observations' at Dun Echt, communicated to the Society in June 1888, also add several other lines which have not previously been recorded.

Previous to Dr. Copeland's observations, the visible spectrum of the Great Nebula was usually given as consisting of four bright lines and a faint continuous spectrum, the wave-lengths of the lines being given as 500, 495, 486, and 434. To these Dr. Copeland added 5874 (D_3), the position of which he had obtained from no less than 33 measurements, and a brightening at 447·6, seen only on one occasion. He found the continuous spectrum extending from 568 to 459·5, and showing "some indications of resolvability into lines or bands."

In September 1888, although the nebula was badly placed for observation, and daybreak interfered, measurements were made at

Hurstside Observatory of four bright lines in the green and blue, the positions being found to agree with those usually assigned. The continuous spectrum was easily seen on widening the slit. On October 13 the sky was exceptionally clear, and the spectrum of the nebula showed six bright lines, five of which had been previously seen by Dr. Copeland, viz. 500, 495, 486, 434, and D_3 (5873), the new line being at 470. The line 5873 was visible with a rather wide slit, and its position was fixed by micrometer readings and by comparison with the sodium (D) lines.

The 470 line was exceedingly faint, and measures were very difficult. The continuous spectrum extended from 460 to 570, and although there were evident irregularities in its intensity no positions of maximum brightness could be fixed. Later observations confirmed this appearance, and on October 27 a decided maximum was noted in the green above the 500 line. This was seen several times afterwards, but not until November 26 could its position be fixed as 520. November 26 and 27 were magnificently clear, especially the latter date, when all the bright lines previously observed were again measured, and in addition a line at 4470, and a maximum brightness of the continuous spectrum near the red end, the position of which was found to be 5592.

The bright lines observed in the nebula in *Orion* were therefore nine in number, their wave-lengths being 5872·6, 5592, 5200, 5001, 4953, 4863, 4703, 4470, 4340·5.

Since the end of November all the lines except 4470 and 5592 have been re-observed, but, considering the exceptional circumstances of November 27, it is not at all surprising that the excessively faint lines have not been visible since that date.

The 5001 line is by far the brightest in the spectrum, and is quite unlike any of the other lines in appearance. It is never seen sharp, but with the narrowest slit always has a fluffy appearance, this being much more marked on the blue than on the red edge. This was most carefully examined for evidence of structure, but the line was always found to be single, and no decided evidence of fluting structure could be made out. It may be that greater dispersion will show structure, but with the dispersion used here no structure could be seen. The F line was a little brighter than the 4953, and these two lines and the fainter hydrogen G (434) were always sharp and comparatively easy to measure. The 5872 is faint, but, being beyond the continuous spectrum, it is not so difficult to see as the 520. A rather wide slit is necessary to see these, but I have frequently seen them both with the slit sufficiently narrow to clearly separate the 5001 and 4953. Of the other lines 4706 has been most frequently seen, but the continuous spectrum and the faintness of the line render the measurements of position difficult. The 4470 and the 5592 were very faint indeed, and have not been seen since November 27, but then their positions were ascertained from four measurements of each.

When the 520 and 5592 were first measured it was thought that they and the 4706 were due to carbon in the nebula. Comparisons were made with the carbon flutings from a spirit-lamp and from a Bunsen's burner. The brightest carbon (517) does not agree with the 520 of the nebula, and the measurements of 520 are so decisive that it is impossible to believe that that brightness is due to the 517 carbon. A more probable explanation of the origin of the 5200 line or brightening is that it is due to magnesium, the 5001 and the 4706 being also due to that metal. It is interesting to note that the 470 line has been observed by Dr. Copeland in a planetary nebula which also contained 500, 495, and F (*Copernicus*, vol. i. p. 2). The 520 has been observed in comet *d* 1880 by Christie, and in comet *c* 1881 by Hasselberg, this line thus furnishing another spectroscopic connection between the comets and nebulæ. The D_3 line has not yet been definitely assigned to any substance, but it and the 559 may be due to manganese radiation; comparisons were tried, using manganese chloride heated in a Bunsen's burner, but, owing to the wide slit and the faintness of the lines in the nebula spectrum, no decisive results could be obtained. The 4863 and 4340 lines are undoubtedly the hydrogen F and G lines. C was searched for with the spectroscope specially adjusted for that line, but no trace of it could be found.

The results of the measurements on those dates when more measures than one were obtained are given in the wave-length table, the figures in brackets indicating the number of measurements made on each occasion. The numbers over the separate columns indicate the relative brightness of the lines; although 8 and 9 are so faint that it is almost impossible to say which is seen with least difficulty.

Table of Measurements of Lines in the Great Nebula in Orion.

Date. 1888.	5	9	6	1	3	2	7	8	4
Sept. 9	...	...	...	5002 (3)	4951 (3)	4863 (3)	...	...	4340 (1)
Oct. 13	5873 (6)	...	...	5000 (8)	4953 (8)	4863 (8)	4695 (3)	...	4340 (7)
Nov. 26	seen (0)	...	5200 (4)	5001 (4)	4954 (4)	4863 (4)	4703 (3)	...	4340 (4)
27	5872 (4)	5592 (4)	5200 (4)	5001 (4)	4952 (4)	4863 (4)	4706 (4)	4470 (4)	4342 (4)
Mean Values	5872·6	5592	5200	5001	4953	4863	4703	4470	4340·5

The continuous spectrum ends between 4703 and 4470 in the blue, and at about 570 in the citron.

The Great Nebula in Andromeda.

The spectrum of this nebula is usually recorded as perfectly continuous, but on October 5 and 9 I noticed in it a decided maximum in the green. Subsequent observations did not confirm this until November 30, when two maxima were visible, and five measures of their positions gave them as 5175 and 548; three measures on December 1 gave 5180 and 5467, and one on December 7 and 9 and two on the 12th gave 5170 and 547 as

the positions. The means, therefore, of the twelve measurements of each maximum show positions of 517·4 and 547·3. No other maximum has been measured, but on one occasion another was suspected down in the blue, although its position could not be fixed.

The Ring Nebula in Lyra.

This nebula has a monochromatic spectrum, and eight measurements at different times give a mean of 5002 as the position of the line. One other line has been suspected but not definitely measured.

Hurstside Observatory,
West Molesey, Surrey.

Spectroscopic Results for the Motions of Stars in the Line of Sight, obtained at the Royal Observatory, Greenwich, in the year 1888. *No. XII.*

(*Communicated by the Astronomer Royal.*)

The results here given are in continuation of those printed in the *Monthly Notices*, vol. xxxvi. p. 318, vol. xxxvii. p. 32, vol. xxxviii. p. 493, vol. xli. p. 109, vol. xlii. p. 230, vol. xliii. p. 81, vol. xliv. p. 89, vol. xlv. p. 330, vol. xlvi. p. 126, vol. xlvii. p. 101, and vol. xlviii. p. 116. The observations were made with the 'half-prism' spectroscope, one 'half-prism,' with a dispersion of about $18\frac{1}{2}°$ from A to H, being used throughout. A magnifying power of 14 was employed.

The cylindrical lens has always been used in front of the slit as in the observations made previously to 1881 and since 1882, March 14. The observations of the Sun, Moon, and planets have been made as a check on the general accuracy of the results.

The day specified in the first column is the civil day, and the hour is that of Greenwich civil time, commencing at Greenwich mean midnight, and reckoning from 0 to 24 hours.

The observations were made by Mr. Maunder throughout.

Motions of Stars in the Line of Sight in Miles per Second, observed with the Half-prism Spectroscope.

(+ denotes Recession; −, Approach).

Date. 1888.		No of Meas.	Line.	Earth's Motion in miles per sec.	Concluded Motion of Star. Meas.	Concluded Motion of Star. Estimd.	Remarks.
				γ Cassiopeiæ.			
Dec.	13 h 20	2	F	+ 8·0	+ 7·1	+ 10·2	Spectrum faint and very tremulous.
				Algol.			
Feb.	1 20	2	F	+ 17·1	− 49·8	− 52·0	Spectrum faint but steady,
Dec.	7 20	2	F	+ 6·4	− 101·8	− 120·0	Spectrum rather faint and tremulous; definition poor.

Date. 1888.	h	No. of Meas.	Line.	Earth's Motion in miles per sec.	Concluded Motion of Star. Meas.	Estimd.	Remarks.
					α Persei.		
Feb.	1 20	4	F	+15·5	−20·9	−16·8	Spectrum bright and steady; definition fair.
Dec.	7 20	2	F	+ 4·4	− 7·4	− 4·4	Spectrum fairly bright and steady; definition of star-line only fair.
					Aldebaran.		
Feb.	1 21	2	F	+16·9	+ 9·6	+21·5	Spectrum fairly bright and steady; star-line narrow and faint.
Dec.	7 23	2	F	+ 2·7	+27·9	+29·1	Spectrum bright and steady; star-line rather faint.
					Capella.		
Dec.	7 21	2	F	− 1·2	+30·2	+28·5	Spectrum bright and steady.
					Rigel.		
Feb.	1 21	4	F	+13·6	+11·0	+17·4	
Mar.	5 21	2	F	+16·0	+22·9	+20·5	Spectrum faint and very tremulous.
Dec.	7 23	2	F	+ 0·3	+25·5	+36·0	Spectrum tremulous but bright.
					β Tauri.		
Feb.	1 22	2	F	+14·7	− 7·7	+ 1·1	
Dec.	7 22	2	F	− 1·6	− 4·1	− 7·5	Star-line diffused but not very broad; spectrum seen well.
					γ Orionis.		
Feb.	1 22	4	F	+14·5	+ 1·5	+ 7·3	
Dec.	7 23	2	F	− 1·0	−25·6	−30·8	Spectrum fairly bright and steady; star-line faint.
					δ Orionis.		
Feb.	1 22	2	F	+13·6	+ 6·3	+14·4	Spectrum steady and fairly bright; star-line very faint.
					ε Orionis.		
Feb.	1 23	2	F	+13·3	−10·9	− 2·8	Spectrum bright and steady; star-line seen fairly well; definition good.
					ζ Orionis.		
Feb.	1 23	4	F	+13·0	−36·1	−41·0	Spectrum bright and fairly steady; star-line faint.
					β Aurigæ.		
Dec.	7 22	2	F	− 3·7	+32·5	+40·1	Spectrum faint, definition fair; star-line very broad and diffused.
					Sirius.		
Feb.	2 0	6	F	+ 7·3	−24·5	−24·5	Spectrum very bright but very unsteady.

Date. 1888.		No. of Meas.	Line.	Earth's Motion in miles per sec.	Concluded Motion of Star. Meas.	Estimd.	Remarks.
				Sirius.			
Mar.	h 5 22	6	F	+12·8	−37·4	−35·8	Spectrum bright but tremulous; star-line seen well.
Dec.	8 0	2	F	− 6·4	+17·9	+20·1	Spectrum bright but very tremulous; star-line very broad, ill-defined and diffused.
				Castor.			
Mar.	6 0	2	F	+15·4	− 6·9	− 5·3	Spectrum bright and steady; star-line dark but very broad and diffused.
	22 0	2	F	+17·4	− 0·4	− 0·6	Spectrum faint but fairly steady; star-line seen fairly well.
				Procyon.			
Mar.	5 23	4	F	+14·0	−17·7	−16·0	Spectrum bright and fairly steady; definition good; star-line seen well.
	21 23	2	F	+16·4	− 3·5	− 2·8	Spectrum faint and tremulous; star-line faint; definition bad; measures made with difficulty.
Dec.	8 0	2	F	−11·2	+25·8	+24·9	Spectrum bright and steady; star-line seen well.
				Pollux.			
Mar.	5 23	2	F	+15·0	−48·0	−47·5	Spectrum bright and steady; star-line faint; measures made with difficulty.
	21 23	2	F	+17·3	−43·3	−40·0	Spectrum faint and tremulous; star-line faint; definition bad; measures made with difficulty.
				Regulus.			
Mar.	22 0	2	F	+10·2	+ 2·9	+ 5·4	Spectrum fairly bright but tremulous; star-line seen fairly well.
May	7 22	2	F	+17·7	− 8·5	− 2·1	Spectrum fairly bright and steady; star-line very ill-defined and diffused.
				β Leonis.			
May	7 23	2	F	+14·9	+46·0	+47·7	Spectrum faint but steady; star-line dark but diffused.
				α Canum Venaticorum.			
May	23 22	2	F	+12·9	−44·7	−45·1	Spectrum fairly bright and steady; star-line well seen.
				Spica.			
May	7 23	2	F	+ 7·7	−31·2	−38·1	Spectrum fairly bright and steady; star-line not very broad, and but little diffused.
	23 23	2	F	+11·7	−38·3	−30·5	Spectrum very tremulous; star-line rather faint; measures made with difficulty.

Date. 1888.			No. of Meas.	Line.	Earth's Motion in miles per sec.	Concluded Motion of Star. Meas.	Estimd.	Remarks.
					Arcturus.			
May	7	h 23	2	F	+ 6·6	−37·5	−48·3	Spectrum bright and steady; star-line narrow but rather faint.
	23	23	2	F	+10·0	−41·0	−36·9	Spectrum very bright but rather tremulous; star-line faint, and seen with difficulty.
					γ Bootis.			
May	24	0	2	F	+ 8·5	−27·4	−27·8	Spectrum bright but rather tremulous; star-line faint and diffused; measures made with great difficulty.
					β Libræ.			
May	24	1	2	F	+ 4·7	−42·8	−42·3	Spectrum rather faint and tremulous; star-line ill-defined, and bisected with difficulty.
					α Coronæ Borealis.			
May	8	0	4	F	+ 1·6	+28·9	+42·4	Spectrum bright and steady; star-line dark and somewhat diffused.
	24	2	2	F	+ 4·9	+21·8	+19·3	Spectrum fairly bright and steady; definition good.
					α Ophiuchi.			
May	8	0	2	F	− 8·0	−24·5	−40·7	Spectrum fairly bright and steady; star-line broad and diffused.
					Vega.			
May	24	2	2	F	− 5·5	−39·3	−37·4	Spectrum bright and steady; star-line well seen; definition good.
					Mars.			
May	7	23	5	F		+ 1·8	+ 2·1	Spectrum bright and steady. Computed motion, +3·7 miles per second.

Moon.

Date.		h	No of Meas.	Line.	Motion Measured.	Remarks.
Feb.	2	0	5	F	−0·3	The coincidence of the two spectra appeared perfect.
Mar.	21	22	5	F	−1·8	
May	24	0	5	F	−2·3	

Sun.

Dec.	10	13	5	F	+ 5·4	Direct comparison showed no perceptible want of coincidence between the two spectra, although the measures all gave the solar F line slightly to the red of the Hβ line.

Observations of Comet e 1888, *made with the Transit Circle at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

Greenwich Mean Solar Time.	Observer.	R.A.	N.P.D. (corrected for Refraction, Parallax and R−D).	Remarks.
1888, Dec. 22 6 34 59	A. P.	0 41 56·66	97 39 34″·19	Cloudy.
26 6 8 30	T.	0 31 9·73	97 33 9·24	Scarcely visible.
31 5 37 44	T.	0 20 1·76	97 20 35·94	Very faint indeed; only a confused mist.
1889, Jan. 1 5 31 51	L.	0 18 4·56	97 17 36·98	Very faint.

The observations were made in a dark field with wires illuminated.

The initials T., L. and A. P., are those of Mr. Thackeray, Mr. Lewis, and Mr. A. Pead.

The observations of this comet, printed in vol. xlix. p. 83, are not corrected for R−D. The corresponding corrections to N.P.D. are for November 27 and December 1 +0″·55; for December 5, 7, and 12, +0″·56.

Observations of Occultations of Stars by the Moon and Phenomena of Jupiter's Satellites, made at the Royal Observatory, Greenwich, in the year 1888.

(*Communicated by the Astronomer Royal.*)

Occultations of Stars by the Moon.

Day of Obs.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
1888, Jan. 28 (*a*)	Disapp. No. 157	S.E. Equat.	220	Eclipsed	10 33 49·94	M.
28	,, 165	Cooke Equat.	200	,,	10 42 32·00	H.
28	,, 180	Lassell Refl.	170	,,	10 52 2·85	H. T.
28	,, 180	Cooke Equat.	200	,,	10 52 3·45	H.
28 (*b*)	,, 181	Lassell Refl.	170	,,	11 1 23·35	H. T.
28	Reapp. 126	Lassell Refl.	170	,,	11 3 33·35	H. T.
28 (*c*)	,, 126	S.E. Equat.	220	,,	11 4(18·74)	M.
28	,, 126	Cooke Equat.	200	,,	11 3 33·12	H.
28 (*d*)	,, 128	Lassell Refl.	170	,,	11 7 (0·35)	H. T.
28 (*e*)	,, 128	Cooke Equat.	200	,,	11 6 11·42	H.
28	Disapp. 201	Lassell Refl.	170	,,	11 27 34·35	H. T.
28 (*f*)	,, 210	Cooke Equat.	200	,,	11 27 32·43	H.
28	Reapp. 157	Lassell Refl.	170	,,	11 34 23·85	H. T.
28	,, 157	Corbett Refr.	110	,,	11 34 24·95	L.
28 (*g*)	,, 150	Lassell Refl.	170	,,	11 37 39·35	H. T.
28	,, 164	Lassell Refl.	170	,,	11 54 3·55	H. T.

Day of Obs.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
1888, Jan. 28 (*h*)	Reapp. 164	S.E. Equat.	200	Eclipsed	11 54 13·64	M.
28	,, 164	Corbett Refr.	110	,,	11 54 4·85	L.
28 (*i*)	,, 165	S.E. Equat.	200	,,	11 55 1·54	M.
28	,, 181	Lassell Refl.	170	,,	11 59 20·35	H. T.
28	,, 180	Lassell Refl.	170	,,	12 4 35·55	H. T.
28 (*j*)	,, 180	S.E. Equat.	200	,,	12 4 42·24	M.
28	,, 180	Corbett Refr.	110	,,	12 4 36·55	L.
May 20	Disapp. *b* Virginis	Altaz.	100	Dark	12 55 33·75	H.
24 (*k*)	,, η Libræ	Altaz.	100	,,	10 51 40·37	A. D.
July 19 (*l*)	,, Piazzi XVI. 236	Altaz.	100	,,	10 26 34·00	T.
23 (*m*)	Reapp. 20 Capricorni	E. Equat.	70	Bright	10 47 7·25	H. T.
Oct. 20	Disapp. μ Ceti	E. Equat.	140	,,	11 38 53·26	A. D.
20 (*n*)	,, μ Ceti	Altaz.	100	,,	11 38 (28·38)	J. P.

The Nos. of the stars observed on the occasion of the total eclipse of the Moon, January 28, are taken from the catalogue of stars printed and circulated by Professor Struve.

Notes.

(*a*) Doubtful if occulted by the Moon or hidden by cloud. (*b*) Observation doubtful.
(*c*) Doubtful if the reappearance was from behind the Moon or cloud. (*d*) Observation doubtful.
(*e*) Observed a bright speck slightly within the Moon's limb; sky clouded over immediately.
(*f*) The star seemed to touch the Moon's limb, and get rapidly fainter, 10^s before it disappeared.
(*g*) Observation doubtful. (*h*) Reappearance instantaneous and well observed.
(*i*) It is believed that the real occultation was observed, although the Moon was partly hidden by cloud at the time.
(*j*) Clouds passing. (*k*) Observation difficult, owing to faintness of the star before disappearance.
(*l*) Star faint. (*m*) Very faint when first seen. (*n*) Observation doubtful.

Phenomena of Jupiter's Satellites.

Day of Obs.	Satellite.	Phenomenon.	Telescope.	Power.	Mean Solar Time of Observation. h m s	Mean Solar Time of N.A. h m s	Observer.
1888, Jan. 5	III.	Ecl. R. First seen	Altaz.	100	17 38 9	17 38 2	H. T.
Apr. 18 (*a*)	III.	Tr. Ing. First contact	E. Equat.	210	16 12 43	16 22	,,
18	III.	Bisection	,,	,,	16 20 13		,,
29 (*b*)	III.	Occ. R. First seen	,,	,,	11 7 58	11 8	L.
29	III.	Last contact	,,	,,	11 15 47		,,
May 5	II.	Ecl. D. Began to fade	Altaz.	100	11 34 35	11 43 33	A. D.
5	II.	Last seen	,,	,,	11 42 33		,,
5 (*c*)	II.	Occ. R. First seen	Lassell Refl.	310	14 51 13	14 55	H. T.
9	I.	Ecl. D. Began to fade	E. Equat.	210	10 54 31	10 59 13	L.
9	I.	Bisection	,,	,,	10 56 26		,,
9	I.	Last seen	,,	,,	10 58 37		,,
10 (*d*)	I.	Tr. Egr. First contact	,,	70	10 32 58	10 33	A. D.
10	I.	Last contact	,,	,,	10 35 48		,,
12	II.	Ecl. D. Began to fade	,,	,,	14 14 40	14 18 42	T.
12	II.	Last seen	,,	,,	14 18 4		,,
24	III.	Tr. Egr. Bisection	Altaz.	100	10 26 5	10 29	A. D.
24	III.	Last contact	,,	,,	10 30 29		,,
31	III.	Tr. Ing. First contact	Lassell Refl.	289	12 4 37	12 12	H. T.
31	III.	Bisection	,,	,,	12 11 52		,,
31	III.	Last contact	,,	,,	12 20 7		,,

Day of Obs.	Satellite.	Phenomenon.	Telescope.	Power.	Mean Solar Time of Observation. h m s	Mean Solar Time of N.A. h m s	Observer.
1888, June 1	I.	Occ. D. First contact	E. Equat.	140	10 47 30		A. D.
1	I.	Bisection	,,	,,	10 51 14	10 52	,,
1	I.	Last seen	,,	,,	10 54 39		,,
2	I.	Tr. Egr. First contact	Altaz.	100	10 8 42	10 12	T.
2	I.	Last contact	,,	,,	10 11 47		,,
6	II.	Occ. D. First contact	E. Equat.	210	10 32 21		,,
6	II.	Bisection	,,	,,	10 35 51	10 36	,,
6	II.	Last seen	,,	,,	10 38 25		,,
24	I.	Occ. D. First contact	,,	,,	10 27 0		A. D.
24	I.	Bisection	,,	,,	10 31 29		,,
24	I.	Last seen	,,	,,	10 35 29	10 34	,,
24	I.	First contact	Lassell Refl.	280	10 26 48		L.
24	I.	Last seen	,,	,,	10 36 3		,,

Notes.

(*a*) Limb of *Jupiter* very tremulous. Daylight rather strong.
(*b*) Observation unsatisfactory; limb of *Jupiter* very badly defined.
(*c*) Observation considered doubtful; *Jupiter* partly hidden by a tree; image extremely bad.
(*d*) Image of *Jupiter* very bad; observation uncertain.

The clear aperture of the mirror of the Lassell Reflector is 24 inches, of the object-glass of the S.E. Equatorial 12·8 inches, of the E. Equatorial 6·7 inches, of the Corbett Refractor $6\frac{1}{2}$ inches, of the Cooke Equatorial 6 inches, and of the Altazimuth $3\frac{3}{4}$ inches.

The initials H. T., A. D., T., M., L., H., and J. P., are those of Mr. Turner, Mr. Downing, Mr. Thackeray, Mr. Maunder, Mr. Lewis, Mr. Hollis, and Mr. Power respectively.

1889, *January* 7.

Étoiles filantes de la période du 9–11 août 1888, *observées en Italie.* Par F. Denza.

(*Communicated by the Secretaries.*)

J'ai achevé la discussion et le calcul des observations faites dans les stations italiennes sur la pluie météorique des *Perseides*, et j'en donne ici un résumé.

Pour rendre les résultats en quelque manière comparables entre eux, je fais suivre une table, dans laquelle se trouvent les résultats obtenus, dans chaque station, pendant les nuits du 9, 10 et 11 août, qui sont les plus importantes de la période, par un seul observateur pendant l'espace d'une heure.

Numéro horaire des étoiles filantes observé dans chaque station pendant les nuits du 9, 10 *et* 11 *août.*

Stations.	Nuits. 9	10	11	Stations.	Nuits. 9	10	11
Recoaro	4	7	1	Doria (Gênes)	36	55	33
Roverbella (Mantoue) ...	11	9	12	Bargone (Emile)	9	9	10
Mantoue	3	15	3	Modène	10	12	10
Crémone	6	6	9	Correggio (Emile) ...	12	13	13
Vocca (Valsesia)	16	33	13	Castel Maggiore ...	25	40	13
Fara (Novare)	24	32	25	Pistoia	35	55	49
Viguarello (Novare) ...	10	28	20	Florence	17	34	30
Aprica (Valteline) ...	9	13	8	Ponte Badia (Florence) ...	9	8	20
S. Giovanni Canarese ...	3	6	5	Fiesole (Florence) ...	9	25	25
Pettinengo (Bielle) ...	18	19	2	Maenza (Rome)	40	25	52
Savillan	9	9	5	S. Giovanni in Galilea ...	14	17	24
Sommariva Perno ...	17	19	17	S. Martino in Pensili ...	15	22	16
Moncalieri	13	18	8	Gaète	16	13	20
Volpeglino (Tortone) ...	24	38	13	Valle di Pompei (Naples)	9	19	9
				Palagonia (Catane) ...	6	13	8

De ce prospectus résulte que les donnés des diverses stations diffèrent beaucoup entre eux. Cette différence a été causée par plusieurs circonstances, telles que la différente transparence du ciel, qui toutefois fut en général serein, la diverse pratique des observateurs, la différente attention usée par ceux-ci dans le tracement des trajectoires, la diversité de l'heure d'observation et aussi le moyen divers dans lequel se manifesta l'apparition. C'est pourquoi nous nous proposons, pour l'année prochaine, de régler ces observations uniformement.

Toutefois, du prospectus que nous avons donné, on voit que, dans la plupart des observatoires, le maximum se vérifia pendant la nuit du 10 au 11. Très petit est le nombre des stations, où le maximum des météores a été observé dans la nuit du 11 au 12. Elles ne font que 4 sur 29, et ce sont 4 aussi celles,

dont les résultats obtenus le 10 et le 11 diffèrent très peu entre eux ; en sorte que nous pouvons affirmer que le maximum de l'apparition fut observé, comme nous l'avons déjà dit, dans la nuit du 10 au 11.

Dans son ensemble la pluie des étoiles filantes a été assez abondante, comparativement à celles des années passées.

La splendeur et la beauté des météores furent plus ou moins rémarquables selon les lieux d'observations. Le radiant principal de la période ne différa que très peu de l'ordinaire, auprès de l'η de *Persée*, et du moyen des divers trajectoires dans plusieurs stations ; sa position résulte en

$$\alpha = 43^\circ,\ \delta = +56^\circ.$$

Comme à l'ordinaire, outre ce radiant principal, on en observa plusieurs autres, surtout dans le *Cygne*, en *Andromède*, dans le *Dragon*, dans l'étoile polaire et ailleurs. Les plus importants sont les suivants :—

1 $-\alpha = 296^\circ$, $\delta = +53^\circ$, près de γ *Cygni*,

2 $-\alpha = 292$, $\delta = +68$, près de δ *Draconis*,

3 $-\alpha = 3$, $\delta = +30$, près de α *Andromedæ*

4 $-\alpha = 15$, $\delta = +89$, près de la *Polaire*.

Plusieurs bolides ont été observés dans nos stations.

De l'Observatoire de Moncalieri :
1888, 24 *Décembre*.

Note on an Error in Mr. Chambers's "Working Catalogue of Red Stars." By Dr. J. Holetschek.

(*Extract from a letter to the Secretary.*)

Permit me to point out an error in the "Working Catalogue of Red Stars," by G. F. Chambers (*Monthly Notices*, vol. xlvii. p. 384).

I have shown in the *Astr. Nachr.* No. 2656 (Band 111, p. 255) that the object given as No. 586 in Birmingham's Catalogue of Red Stars does not exist in the heavens, but has only been placed in that catalogue in consequence of a mistake of Secchi's. Birmingham has taken the place of the star in question from Secchi's "Prodromo di un Catalogo fisico delle stelle colorate," in which this object bears the number 403. We find in Secchi's paper, "Sugli spettri prismatici delle stelle fisse," Memoria II., which was evidently made use of in the compilation of the "Prodromo," the following particulars on page 58 :

Schj. No. 253 $\alpha = 21^h\ 30{\cdot}2^m$ $\delta = +58^\circ\ 8'$ var.

This is, however, as may be seen from a glance at Schjellerup's "Catalog der rothen isolirten Sterne" (*Astr. Nachr.* No. 1591), nothing else than W. Herschel's "Garnet Star," but the minute of right ascension should be 39·2, instead of 30·2.

Secchi has, however, given in the "Prodromo," not only the correct, but also the incorrect position, and thus out of *one* red star has made *two*. The object Secchi No. 403=Birmingham No. 586=Chambers No. 648 should therefore be struck out.

Vienna: 1889, *Jan.* 14.

Ephemeris for Physical Observations of the Moon. By A. Marth.

1889, *April* 1 *to June* 30.

Greenwich Noon.	Selenographical Colong. of the Sun.	Lat. of the Sun.	Geocentric Libration. Long. of the Earth.	Lat. of the Earth.	Amount.	Direction.
	°	°	°	°	°	°
April 1	277·98	−1·52	+3·98	+6·49	7·61	328·6
2	290·20	1·52	3·31	6·17	7·00	331·9
3	302·42	1·52	2·40	5·57	6·06	336·8
4	314·64	1·52	+1·28	4·72	4·89	344·9
5	326·85	1·53	0·00	3·68	3·68	0·1
6	339·05	1·53	−1·39	2·48	2·84	29·1
7	351·25	−1·53	−2·79	+1·18	3·02	67·0
8	3·45	1·54	4·11	−0·18	4·12	92·5
9	15·64	1·54	5·26	1·56	5·49	106·5
10	27·83	1·54	6·16	2·89	6·80	115·2
11	40·01	1·54	6·70	4·11	7·86	121·7
12	52·18	1·54	6·81	5·17	8·54	127·3
13	64·35	1·54	6·47	5·97	8·79	132·8
14	76·52	−1·53	−5·68	−6·44	8·58	138·8
15	88·69	1·53	4·49	6·53	7·92	145·6
16	100 85	1·52	3 02	6·21	6·90	154·1
17	113·02	1·52	−1·40	5·46	5·64	165·7
18	125·19	1·51	+0·23	4·35	4·35	183·1
19	137·37	1·50	1·74	2·95	3·43	210·5
20	149·55	1·49	3·04	−1·38	3·34	241·6
21	161·74	−1·48	+4·09	+0·26	4·09	273·6
22	173·94	1·47	4·85	1·85	5·18	290 9
23	186·14	1·46	5·35	3·30	6·28	301·7
24	198·35	1·45	5·61	4·54	7·21	309·1
25	210·57	1·43	5·65	5·51	7·89	314·4
26	222·79	1·42	5·50	6·18	8·26	318·5
27	235·02	1·41	5·14	6·53	8·30	321·9
28	247·25	−1·40	+4·62	+6·55	8·01	325·0
29	259·49	1·39	3·90	6·26	7·37	328·2
30	271·73	1·38	2·99	5·68	6·42	332·3

Greenwich Noon.	Selenographical Colong. of the Sun.	Selenographical Lat. of the Sun.	Geocentric Libration. Long. of the Earth.	Geocentric Libration. Lat. of the Earth.	Geocentric Libration. Amount.	Geocentric Libration. Direction.
	°	°	°	°	°	°
May 1	283·96	1·37	1·90	4·85	5·21	338·6
2	296·19	1·36	+0·67	3·82	3·88	350 0
3	308·42	1·35	−0·68	2·62	2·71	14·5
4	320·65	1·34	2·10	+1·32	2·48	57·8
5	332·88	−1·33	−3·52	−0·04	3·54	96·7
6	345·10	1·32	4·85	1·41	5·06	106·3
7	357·31	1·31	6·02	2·74	6·62	114·5
8	9·52	1·30	6·93	3·97	7·99	119·9
9	21·73	1·29	7·50	5·05	9·03	124·1
10	33·93	1·27	7·63	5·90	9·63	127·9
11	46·12	1·26	7·28	6·45	9·72	131·8
12	58·31	−1·24	−6·46	−6·66	9·27	136·0
13	70·49	1·22	5·18	6·45	8·26	141·4
14	82·67	1·20	3·54	5·81	6·80	148·8
15	94·85	1·18	−1·67	4·76	5·05	160·8
16	107·03	1·16	+0·26	3·38	3·39	184·5
17	119·21	1·14	2·10	1·76	2·74	230 0
18	131·40	1·12	3·72	−0·05	3·72	269·3
19	143·59	−1·10	+5·04	+1·64	5·30	288·1
20	155·79	1·08	5·99	3·19	6·78	298·1
21	168·00	1·05	6·59	4·51	7 98	304·5
22	180·22	1·03	6·85	5·54	8·80	309·1
23	192·44	1·01	6·79	6·25	9·22	312·8
24	204·67	0·99	6·46	6·64	9·25	315·9
25	216·90	0·97	5·89	6·69	8·90	318·8
26	229·14	−0·95	+5·11	+6·43	8·20	321·7
27	241·38	0·93	4·14	5 88	7·19	325·0
28	253·62	0·91	3·02	5·07	5·90	329 3
29	265·87	0·89	1·77	4·05	4·42	336·4
30	278·12	0 87	+0·42	2·86	2·89	351·6
31	290·36	0·85	−0·98	1·54	1·83	32·5
June 1	302·61	0·83	2·41	+0·16	2·42	86·1
2	314·85	−0·81	−3·80	−1·23	4·00	107 9
3	327·09	0·79	5·10	2·58	5·71	116 9
4	339·32	0·77	6·22	3·84	7·31	121·8
5	351·55	0·75	7·11	4·94	8·65	124·9
6	3 77	0·73	7·69	5·84	9·64	127·4
7	15·99	0·71	7·86	6·46	10·16	129·6

Greenwich Noon.	Selenographical Colong. of the Sun.	Lat. of the Sun.	Geocentric Libration. Long. of the Earth.	Lat. of the Earth.	Amount.	Direction.
	°	°	°	°	°	°
June 8	28·20	0·68	7·60	6·77	10·17	131·9
9	40·41	−0·66	−6·88	−6·69	9·59	134·8
10	52·61	0·63	5·71	6·20	8·42	137·5
11	64·80	0·60	4·15	5·30	6·72	142·0
12	76·99	0·57	2·30	4·02	4·63	150·3
13	89·18	0·54	−0·30	2·44	2·45	173·1
14	101·37	0·52	+1·69	−0·67	1·82	248·5
15	113·56	0·49	3·53	+1·11	3·70	287·4
16	125·75	−0·46	+5·09	+2·79	5·80	298·8
17	137·95	0·43	6·29	4·25	7·58	304·2
18	150·16	0·40	7·08	5·41	8·90	307·5
19	162·37	0·37	7·46	6·23	9·71	310·1
20	174·59	0·34	7·45	6·70	10·00	312·2
21	186·82	0·32	7·08	6·81	9·81	314·1
22	199·05	0·29	6·40	6·60	9·19	316·1
23	211·28	−0·27	+5·48	+6·10	8·19	318·2
24	223·52	0·24	4·35	5·32	6·87	320·8
25	235·77	0·21	3·08	4·33	5·31	324·6
26	248·02	0·19	1·72	3·15	3·59	331·3
27	260·27	0·17	+0·31	1·84	1·87	350·3
28	272·52	0·15	−1·10	+0·46	1·19	67·4
29	284·77	0·12	2·48	−0·95	2·65	111·1
30	297·02	−0·10	−3·77	−2·33	4·43	121·8

With the ephemeris for physical observations of the Moon for the second half of the year I intend to give a list of published lunar sketches, arranged in the order in which the Sun reaches the same altitudes above the horizon of the spots represented as at the times assigned to the sketches. These times ought to be those when the outlines of the shadows are drawn, if the evidence afforded by the sketches is not to be weakened. In the list only such sketches can be inserted as are at least approximately timed, and a number of merely dated sketches will have to be left out if such approximate times, and also the meridian to which they refer, are not made known to me.

Ephemeris of the Satellites of Uranus, 1889. By A. Marth.

Greenwich Noon. 1889.	P	*Ariel.* a_1	b_1	u_1-U	*Umbriel.* a_2	b_2	u_2-U
	°	″	″	°	″	″	°
Feb. 26	282·643	14·97	+ 8·59	168·62	20·85	+ 11·97	197·03
Mar. 8	·718	15·06	8·58	157·00	20·97	11·95	345·71
18	·803	15·13	8·54	145·34	21·07	11·90	134·37
28	·896	15·17	8·48	133·66	21·14	11·81	283·00
Apr. 7	282·992	15·19	8·40	121·96	21·17	11·70	71·62
17	283·085	15·19	8·30	110·23	21·16	11·56	220·22
27	·173	15·16	8·19	98·48	21·12	11·41	8·82
May 7	·252	15·10	+ 8·08	86 71	21·04	+ 11·25	157·41
17	·319	15·02	7·96	74·93	20·93	11·09	305·99
27	·372	14·93	7·85	63·15	20·79	10·94	94·58
June 6	·409	14·82	7·75	51·36	20·64	10·80	243·17
16	·430	14·69	7·66	39·58	20·47	10·67	31·77
26	·433	14·56	7·58	27·81	20·29	10·57	180·38
July 6	·418	14·43	7·53	16·04	20·10	10·49	329·00
16	283·386	14·30	+ 7·49	4·29	19·92	+ 10·43	117·64

Greenwich Noon. 1889.	*Titania.* a_3	b_3	u_3-U	*Oberon.* a_4	b_4	u_4-U	U	B
	″	″	°	″	″	°	°	°
Feb. 26	34·20	+ 19·63	164·71	45·73	+ 26·25	321·69	0·790	+ 35·02
Mar. 8	34·40	19·60	218·19	46·01	26·21	229·03	0·853	34·72
18	34·57	19·51	271·65	46·22	26·09	136·37	0·925	34·37
28	34·67	19·37	325·09	46·37	25·91	43·69	1·004	33·97
Apr. 7	34·72	19·19	18·53	46·43	25·66	311·01	1·085	33·55
17	34·71	18·97	71·96	46·41	25·37	218·32	1·165	33·13
27	34·63	18·72	125·39	46·32	25·04	125·63	1·240	32·72
May 7	34·51	+ 18·46	178·82	46·15	+ 24·69	32·95	1·307	+ 32·34
17	34·33	18·20	232·25	45·91	24·34	300·27	1·363	32·01
27	34·11	17·94	285·70	45·61	24·00	207·60	1·407	31·74
June 6	33·85	17·71	339·15	45·27	23·68	114·95	1·437	31·54
16	33·57	17·50	32·61	44·90	23·41	22·31	1·452	31·42
26	33·28	17·33	86·08	44·50	23·18	289·68	1·451	31·38
July 6	32·98	17·20	139·57	44·10	23·00	197·06	1·433	31·43
16	32·68	+ 17·11	193·08	43·70	+ 22·88	104·46	1·399	+ 31·57

P, angle of position of the minor axes of the apparent orbits.
a, b, major and minor semi-axes of the apparent orbits.

$u-U$, orbital longitudes of the satellites reckoned from the points which are in superior conjunction with the planet or in opposition to the Earth.

$U+180°$, planeto-centric longitude of the Earth reckoned in the assumed plane of the orbits from the ascending node on the celestial equator.

B, latitude of the Earth above the plane of the orbits, the ascending node N and inclination J, of which, in reference to the equator of 1880·0, are assumed to have the values

$$N = 165°{\cdot}770; \quad J = 75°{\cdot}210, \text{ or}$$

$$\text{for } 1889{\cdot}0 \quad 165°{\cdot}898 \quad 75°{\cdot}198.$$

The values of P, a, b, and $u-U$ are to be interpolated directly for the times for which the apparent positions are required, the equation of light being already taken into account. The degrees of the differences of successive values of $u-U$ are in the case of *Ar.* 1428°, *Um.* 868°, *Tit.* 413°, *Ob.* 267°. The position-angles p and distances s are then found by means of the formulæ

$$s \sin (p-P) = a \sin (u-U),$$

$$s \cos (p-P) = b \cos (u-U).$$

The satellites will be at their greatest elongations (N. in position-angle $P+90°$, S. in position $P-90°$) and at their conjunctions (superior conjunction in position-angle P, inferior conjunction in position $P-180°$) with the planet's centre at the following hours, Greenwich mean times:—

Ariel.

	N. Elong.	S. Elong.		N. Elong.	S. Elong.		N. Elong.	S. Elong.
1889.	h	h		h	h		h	h
Feb. 27	23·3	M. 1 5·5	Apr. 9	7·1	10 13·4	May 19	15·0	20 21·3
Mar. 2	11·8	3 18·0	11	19·6	13 1·9	22	3·5	23 9·8
5	0·3	6 6·5	14	8·1	15 14·4	24	16·0	25 22·3
7	12·7	8 19·0	16	20·6	18 2·8	27	4·5	28 10·8
10	1·2	11 7·5	19	9·1	20 15·3	29	17·0	30 23·3
12	13·7	13 20·0	21	21·6	23 3·8	June 1	5·5	2 11·7
15	2·2	16 8·5	24	10·1	25 16·3	3	18·0	5 0·2
17	14·7	18 20·9	26	22·6	28 4·8	6	6·5	7 12·7
20	3·2	21 9·4	29	11·1	30 17·3	8	19·0	10 1·2
22	15·7	23 21·9	May 1	23·6	3 5·8	11	7·5	12 13·7
25	4·2	26 10·4	4	12·1	5 18·3	13	20·0	15 2·2
27	16·7	28 22·9	7	0·6	8 6·8	16	8·5	17 14·7
30	5·2	31 11·4	9	13·0	10 19·3	18	21·0	20 3·2
Apr. 1	17·6	2 23·9	12	1·5	13 7·8	21	9·5	22 15·7
4	6·1	5 12·4	14	14·0	15 20·3	23	22·0	25 4·2
6	18·6	8 0·9	17	2·5	18 8·8	26	10·4	27 16·7

Umbriel.

1889.	N. Elong. h	S. Elong. h		N. Elong. h	S. Elong. h		N. Elong. h	S. Elong. h
Feb. 28	21·9	M. 2 23·6	Apr. 11	8·6	13 10·3	May 22	19·3	24 21·0
Mar. 5	1·3	7 3·1	15	12·0	17 13·8	26	22·7	29 0·5
9	4·8	11 6·5	19	15·5	21 17·2	31	2·2	J. 2 3·9
13	8·3	15 10·0	23	19·0	25 20·7	June 4	5·7	6 7·4
17	11·7	19 13·5	27	22·4	30 0·2	8	9·1	10 10·9
21	15·2	23 16·9	May 2	1·9	4 3·6	12	12·6	14 14·3
25	18·7	27 20·4	6	5·4	8 7·1	16	16·1	18 17·8
29	22·1	31 23·9	10	8·8	12 10·6	20	19·6	22 21·3
Apr. 3	1·6	5 3·3	14	12·3	16 14·1	24	23·0	27 0·8
7	5·1	9 6·8	18	15·8	20 17·5	29	2·5	Jy. 1 4·2

Titania.

	h			h			h			h
Feb. 26	8·9	inf.	Apr. 2	4·8		May 7	0·7	inf.	June 10	20·6
28	13·1	S.	4	9·0		9	4·9	S.	13	0·8
Mar. 2	17·3	sup.	6	13·2		11	9·2	sup.	15	5·1
4	21·6	N.	8	17·5		13	13·4	N.	17	9·3
7	1·8	inf.	10	21·7		15	17·7	inf.	19	13·6
9	6·1	S.	13	2·0		17	21·9	S.	21	17·8
11	10·3	sup.	15	6·2		20	2·1	sup.	23	22·0
13	14·6	N.	17	10·5		22	6·4	N.	26	2·3
15	18·8	inf.	19	14·7		24	10·6	inf.	28	6·5
17	23·0	S.	21	19·0		26	14·9	S.	30	10·7
20	3·3	sup.	23	23·2		28	19·1	sup.	July 2	15·0
22	7·5	N.	26	3·5		30	23·4	N.	4	19·2
24	11·8	inf.	28	7·7	June	2	3·6	inf.	6	23·5
26	16·0	S.	30	11·9		4	7·9	S.	9	3·7
28	20·3	sup.	May 2	16·2		6	12·1	sup.	11	7·9
31	0·5	N.	4	20·4		8	16·4	N.	13	12·2

Oberon.

	h			h			h			h
Feb. 27	10·4	sup.	Apr. 8	20·0		May 19	5·6	sup.	June 28	15·1
Mar. 2	19·2	N.	12	4·8		22	14·4	N.	July 1	23·9
6	4·0	inf.	15	13·6		25	23·2	inf.	5	8·7
9	12·8	S.	18	22·4		29	8·0	S.	8	17·5
12	21·6	sup.	22	7·2	June	1	16·8	sup.	12	2·4
16	6·4	N.	25	16·0		5	1·6	N.	15	11·2
19	15·2	inf.	29	0·8		8	10·4	inf.		
23	0·0	S.	May 2	9·6		11	19·2	S.		
26	8·8	sup.	5	18·4		15	4·0	sup.		
29	17·6	N.	9	3·2		18	12·8	N.		
Apr. 2	2·4	inf.	12	12·0		21	21·5	inf.		
5	11·2	S.	15	20·8		25	6·3	S.		

Titania and *Oberon* will appear in the same direction from the centre of the planet:—

		h			h			h
Mar.	8	17·8	Apr.	27	0·4	June	15	7·1
Apr.	2	9·1	May	21	15·7	July	9	22·4

and in opposite directions:

		h			h			h
Feb.	24	10·2	Apr.	14	16·7	June	2	23·4
Mar.	21	1·5	May	9	8·1		27	14·8